Sirius
Mathematics Center

O16w: Geometry and Homological Mirror Symmetry

December 11-14 | 2021

Sirius University of Science and Technology
Sirius Mathematics Center
Higher School of Economics
International Laboratory for Mirror Symmetry and Automorphic Forms

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International Conference

Geometry and Homological Mirror Symmetry

December 11–14, 2021

Program and Abstracts

Sochi, 2021

Organizers:

Ludmil Katzarkov

Victor Przyjalkowski

Ivan Cheltsov

The Conference is held at the Sirius Mathematics Center.

Geometry and Homological Mirror Symmetry:
https://ms.hse.ru/en/siriusdec2021

CONFERENCE PROGRAM

DECEMBER 11, SATURDAY

10^{00} Ilia Zharkov, *Lagrangian SYZ fibrations.*

11^{15} Bin Wang, *A Noether–Lefschetz Theorem for Spectral Varieties with Applications.*

LUNCH

14^{00} Vladimir Baranovsky, *Quantization of sheaves and applications.*

15^{15} Christopher Brav, *Calabi–Yau categories with boundary and their symmetries.*

COFFEE BREAK

16^{45} Antoni Rangachev (zoom talk), *Rigidity and generalized smoothability.*

18^{00} RECEPTION

DECEMBER 12, SUNDAY

10^{00} Stefan Ivanov, *Solution of the qc Yamabe equation on a 3-Sasakian manifold, extremals of the Sobolev-Folland-Stein inequality on the quaternionic Heisenberg groups and the qc Yamabe problem.*

11^{15} Alexander Petkov, *The Yamabe problem on non-spherical quaternionic contact manifolds.*

LUNCH

14^{00} Konstantin Loginov, *On K-stability of three-dimensional log Fano varieties.*

COFFEE BREAK

15^{30} Velichka Milousheva, *Minimal Lorentz surfaces in pseudo-Euclidean 4-space with neutral metric and their canonical Weierstrass representation.*

16^{45} Vadim Vologodsky, *Quantization of symplectic varieties over the ring of integers.*

18^{00} Andrew Harder (zoom talk), *Hodge structures and hybrid Landau–Ginzburg models.*

DECEMBER 13, MONDAY

10^{30} Grigory Belousov, *Cylinders in del Pezzo surfaces of degree two.*

12^{00} Maxim Kontsevich (zoom talk), *Boutroux curves and generalizations.*

LUNCH

15^{00} Alexey Golota, *Jordan property for groups of bimeromorphic automorphisms.*

15^{30} Lyalya Guseva, *On the Derived Category of the Cayley Grassmannian.*

COFFEE BREAK

16^{30} Mikhail Ovcharenko, *Dolgachev–Nikulin Duality for Fibers of Toric Landau–Ginzburg Models of Smooth Fano Threefolds.*

17^{00} Vasily Rozhdestvensky, *Steenrod's problem on realization of cycles. Old and new.*

17^{30} Denis Auroux (zoom talk), *Lagrangian Floer theory for trivalent graphs and HMS for curves.*

DECEMBER 14, TUESDAY

10^{00} Mentzelos Melistas, *Reduction of Abelian varieties with torsion points.*

11^{15} Dmitry Krekov, *Two-dimensional height pairing.*

11^{45} Igor Spiridonov, *On the homology of Torelli groups.*

12^{15} Andrey Konovalov, *Topological K-theory of dg-algebras and the lattice conjecture.*

LUNCH

15^{15} Alexander Efimov (zoom talk), *Mittag-Leffler inverse systems of DG categories.*

COFFEE BREAK

16^{30} Charles Doran (zoom talk), *The Mirror Clemens-Schmid Sequence.*

Abstracts of the talks

Lagrangian Floer theory for trivalent graphs and HMS for curves

Denis Auroux

Harvard University

The mirror of a genus g curve can be viewed as a trivalent configuration of $3g-3$ rational curves meeting in $2g-2$ triple points; more precisely, this singular configuration arises as the critical locus of the superpotential in a 3-dimensional Landau–Ginzburg mirror. In joint work with Alexander Efimov and Ludmil Katzarkov, we introduce a notion of Fukaya category for such a configuration of rational curves, where objects are embedded graphs with trivalent vertices at the triple points, and morphisms are linear combinations of intersection points as in usual Floer theory. We will describe the construction of the structure maps of these Fukaya categories, attempt to provide some motivation, and outline examples of calculations that can be carried out to verify homological mirror symmetry in this setting.

Quantization of sheaves and applications

Vladimir Baranovsky

University of California - Irvine

I will give an overview of quantization of coherent sheaves on an algebraic symplectic variety, and outline some open questions in the area. One of the applications is a generalization of a theorem, due to Etingof and Schedler, about finite dimensional simple modules over an algebra of quantized functions on a symplectic resolution of singularities.

Cylinders in del Pezzo surfaces of degree two

Grigory Belousov

Bauman Moscow State Technical University

We will consider the problem of the existence of cylinders in del Pezzo surfaces of degree two with Du Val singularities. We will construct cylinders in singular del Pezzo surfaces. Then we will give a sketch of the proof of absence of some cylinders in smooth del Pezzo surfaces.

Calabi-Yau categories with boundary and their symmetries

Christopher Brav

Higher School of Economics

We review the notion of relative Calabi-Yau structure on a dg functor (a non-commutative analogue of an oriented manifold with boundary introduced in joint work with Toby Dyckerhoff), explain how the Lie algebra of its automorphism group is a chain-level generalisation of the string Lie algebra of Chas-Sullivan and the necklace Lie algebra of Kontsevich, and discuss some examples and applications (part of joint work Nick Rozenblyum).

The Mirror Clemens-Schmid Sequence

Charles Doran

University of Alberta

I will present a four-term exact sequence relating the cohomology of a fibration to the cohomology of an open set obtained by removing the preimage of a general linear section of the base. This exact sequence respects three filtrations, the Hodge, weight, and perverse Leray filtrations, so that it is an exact sequence of mixed Hodge structures on the graded pieces of the perverse Leray filtration. I claim that this sequence should be thought of as a mirror to the Clemens–Schmid sequence describing the structure of a degeneration and formulate a 'mirror $P = W$' conjecture relating the filtrations on each side. Finally, I will present evidence for this conjecture coming from the K3 surface setting. This is joint work with Alan Thompson (arXiv:2109.04849).

Mittag-Leffler inverse systems of DG categories

Alexander Efimov

Steklov Mathematical Institute & Higher School of Economics

We will introduce a certain class of sufficiently nice inverse systems of DG categories, which we call (secondary) Mittag-Leffler systems. This notion covers both the context of formal schemes and its more general non-commutative version: functors from a proper DG category to some dualizable presentable DG category. We expect that for ML systems the K-theory commutes with inverse limits in an appropriate sense.

Jordan property for groups of bimeromorphic automorphisms

Alexey Golota

Higher School of Economics

We say that a group G has Jordan property if all finite subgroups of G can be obtained as extensions of groups from a finite list by finite abelian groups. In my talk I will survey some recent results related to Jordan property for groups of bimeromorphic automorphisms of compact Kaehler manifolds.

Gauss–Manin connection and higher local fields

Sergey Gorchinskiy

Steklov Mathematical Institute & Higher School of Economics

The talk is based on a joint project with Victor Przyjalkowski. We show how higher local fields and Parshin's theory of higher residues allows one to find explicitly solutions of Picard–Fuchs equation or, more generally, horizontal vectors with respect to Gauss–Manin connection locally at singular values of morphisms. As a particular case for morphisms defined by Laurent polynomials, we obtain the famous main period formula used in Mirror Symmetry.

On the Derived Category of the Cayley Grassmannian

Lyalya Guseva

Higher School of Economics

We will describe a full exceptional collection of vector bundles in the bounded derived category of coherent sheaves on the Cayley Grassmannian, that parametrizes four-dimensional subalgebras of the complexified octonions.

Hodge structures and hybrid Landau–Ginzburg models

Andrew Harder

Lehigh University

A hybrid Landau-Ginzburg model is a collection of data consisting of a (smooth) quasiprojective variety Y along with regular functions $f_1, \dots, f_k$ on Y. Such data appears naturally in mirror symmetry as the mirror to a log Calabi–Yau pair (X, D) where D is a divisor with nef components $D_1, \dots, D_k$.

Given a pair (Y, f) (i.e. the case where $k = 1$), Shamoto, following Katzarkov–Kontsevich–Pantev and separate work of Sabbah, constructed a mixed Hodge structure which encodes both data about the global topology of Y and the limiting mixed Hodge structure on fibres of f near infinity. I'll discuss work in progress which extends Shamoto's construction to arbitrary collections of potential functions, and proves that in convenient cases the mixed Hodge structure attached to $(Y, f_1, \dots, f_k)$ agrees with Shamoto's mixed Hodge structure for $(Y, f_1 + \cdots + f_k)$. This is, in some sense, a global version of a classical theorem of Sebastiani–Thom.

We will then apply this to the following problem. Attached to a Fano complete intersection X in a toric variety, there are two constructions of the mirror Landau–Ginzburg model which are of different dimensions. We use the result discussed in the previous paragraph to show that these two Landau–Ginzburg models carry the same mixed Hodge structure as predicted by mirror symmetry.

Solution of the qc Yamabe equation on a 3-Sasakian manifold, extremals of the Sobolev-Folland-Stein inequality on the quaternionic Heisenberg groups and the qc Yamabe problem

Stefan Ivanov

Sofia University

A complete solution to the quaternionic contact Yamabe equation on the 4n+3-dimensional sphere as well as on the quaternionic Heisenberg group is given and a uniqueness result for a compact locally 3-Sasakian manifold is shown. Consequently, the best constant and all extremals of the Sobolev-Folland-Stein inequality on the quaternionic Heisenberg group are determined. The quaternionic contact Yamabe problem is solved on compact non-locally spherical quaternionic contact manifolds.

Topological K-theory of dg-algebras and the lattice conjecture

Andrey Konovalov

Higher School of Economics

I will discuss the problem of constructing a natural rational structure on periodic cyclic homology of dg-algebras and dg-categories. The promising candidate is topological K-theory of dg-categories defined by A. Blanc. I will show that (semi-)topological K-theory is a derived nilpotent (or, equivalently, truncating) invariant and that it provides periodic homology with a rational structure in a number of examples.

Boutroux curves and generalizations

Maxim Kontsevich

Institut des Hautes Études Scientifiques

Let C be a smooth complex algebraic curve. A closed curve in the cotangent bundle to C (i.e. the graph of a multivalued 1-form) is called a Boutroux curve iff all the periods of the restriction of the Liouville form are purely imaginary. In the case square roots of quadratic differentials, such curves are the same as Jenkins-Strebel differentials.

I'll review several conjectures of Soibelman and myself concerning Boutroux curves from the point of view of wall-crossings theory. Also, I will speak about a higher-dimensional generalization, involving Fukaya categories for certain pencils of real symplectic forms.

Two-dimensional height pairing

Dmitry Krekov

Higher School of Economics

Consider a d-dimensional smooth projective variety X over an arbitrary field K. One can define an intersection index which is a bilinear pairing between Chow groups of cycles on X of codimension i and $d-i$.

If K is a function field or a number field, Beilinson and Bloch defined (under certain conjectural assumptions in the latter case) a height pairing, which is a bilinear pairing between subgroups of homologically trivial cycles in i-th and $(d-i+1)$-th Chow groups of X. Both pairings are well-behaved with respect to correspondences.

I will define a class of fields and construct a pairing between certain functorial subgroups of Chow groups of cycles of codimension i and $d-i+2$ of certain class of varieties over such fields. This pairing is similarly well-behaved with respect to correspondences. If time permits I will provide some examples of computation of this pairing.

On K-stability of three-dimensional log Fano varieties

Konstantin Loginov

Higher School of Economics

K-stability is an algebraic invariant which characterises the existence of a Kahler-Einstein metrics on Fano varieties. Initially, this invariant was defined in terms of all one-parametric degenerations of a given variety. Later, K. Fujita proposed a valuative criterion of K-stability which allows to check it in terms of numerical invariants of divisors over given variety. Using this criterion and the inductive approach by Abban-Zhuang, a group of 9 authors solved the problem of characterisation of K-stable varieties for a general element in each of 105 families of smooth Fano threefolds. I will consider an analogous problem for three-dimensional log Fano varieties, that is, pairs (X, D) with boundary divisor D, such that $-K_X - D$ is ample. I will concentrate on so-called log Fano pairs of Maeda type, introduced by Fujita. To solve this problem, we will develop a generalisation of Abban-Zhuang theory, as well as the classification of log smooth log Fano pairs in dimension 3 due to Maeda.

Reduction of Abelian varieties with torsion points

Mentzelos Melistas

Steklov Mathematical Institute

Let R be a discrete valuation ring with fraction field K of characteristic 0 and algebraically closed residue field of characteristic p¿0. In this talk, reduction properties of Abelian varieties which are defined over K and have a K-rational point of order p will be discussed. Time permitting, we will also discuss the interplay between torsion and Tamagawa numbers of Abelian varieties defined over number fields.

Minimal Lorentz surfaces in pseudo-Euclidean 4-space with neutral metric and their canonical Weierstrass representation

Velichka Milousheva

Institute of Mathematics & Informatics, Bulgarian Academy of Sciences

The minimal Lorentz surfaces in the pseudo-Euclidean 4-space with neutral metric $\mathbb{R}^4_2$ whose first normal space is two-dimensional and whose Gauss curvature K and normal curvature $\varkappa$ satisfy the condition $K^2 - \varkappa^2 \neq 0$ are called minimal Lorentz surfaces of general type. These surfaces admit special isothermal parameters, called canonical. The Gauss curvature K and the normal curvature $\varkappa$ of such a surface considered as functions of the canonical parameters satisfy the following system of natural PDEs:

$$\begin{aligned} \sqrt[4]{\left|K^2-\varkappa^2\right|}\,\Delta^h \ln\left|K^2-\varkappa^2\right| &= 8K; \\ \sqrt[4]{\left|K^2-\varkappa^2\right|}\,\Delta^h \ln\left|\tfrac{K+\varkappa}{K-\varkappa}\right| &= 4\varkappa; \end{aligned} \qquad K^2-\varkappa^2 \neq 0. \tag{1}$$

We find a Weierstrass representation with respect to isothermal parameters of any minimal Lorentz surface of general type. Further, we obtain a Weierstrass representation with respect to canonical parameters which describes locally all these surfaces in terms of four real functions. Using the canonical Weierstrass representation we solve explicitly the system (1) of natural PDEs expressing any solution to this system by means of four real functions of one variable. Finally, by means of the canonical Weierstrass representation formula we give examples of minimal Lorentz surfaces of general type in $\mathbb{R}^4_2$ parametrized by canonical parameters.

Dolgachev–Nikulin Duality for Fibers of Toric Landau–Ginzburg Models of Smooth Fano Threefolds

Mikhail Ovcharenko

Higher School of Economics

Mirror Symmetry corresponds to Fano varieties certain one-dimensional families which are called Landau–Ginzburg models. Elements of these families are expected to be Calabi–Yau varieties mirror dual to anticanonical sections of Fano varieties. In partucular, in the three-dimensional case we deal with Mirror Symmetry of K3 surfaces. One of the most interesting forms of Mirror Symmetry for K3 surfaces is so called Dolgachev–Nikulin duality: basically, it interchanges the lattices of algebraic and transcendental cycles on a K3 surface.

Theory of toric Landau–Ginzburg models provides an effective method of constructing Landau–Ginzburg models of Fano varieties. Given a Fano variety, one can find its toric Landau–Ginzburg model, that is a Laurent polynomial which periods correspond in a certain way to Gromov–Witten theory of the Fano variety. This Laurent polynomial should also correspond to a toric degeneration of the Fano variety and admit a log Calabi-Yau compactification: this compactification gives a family over projective line whose anticanonical class is a fiber.

It is natural to expect that Dolgachev–Nikulin duality holds for toric Landau–Ginzburg models of smooth Fano threefolds. This conjecture was proved by Ilten–Lewis–Przyjalkowski in the case of Picard rank 1. Our main result is the explicit construction of "the expected Picard lattices" of generic fibers of the Landau–Ginzburg models. Namely, for each case in Iskovskikh–Mori–Mukai classification of smooth Fano threefolds we have constructed the sublattice in the Picard lattice of a generic fiber of the toric Landau–Ginzburg model which is orthogonal to the primitive embedding of the direct sum of a hyperbolic plane and the Picard lattice of a smooth Fano threefold into the second integral cohomology lattice of a general fiber. Such an orthogonal decomposition is a form of Dolgachev–Nikulin duality.

We will also discuss the possible approach to the proof of the coincidence of "the expected Picard lattice" with the actual Picard lattice of a generic fiber.

The Yamabe problem on non-spherical quaternionic contact manifolds

Alexander Petkov

Sofia University

We show that the Yamabe problem in quaternionic contact (qc) geometry has a solution on any compact qc manifold which is non-locally qc equivalent to the standard 3-Sasakian sphere. Precisely, we show that on a compact non-locally spherical qc manifold there exists a qc conformal qc structure with constant qc scalar curvature.

Rigidity and generalized smoothability

Antoni Rangachev

University of Chicago

I will introduce a class of singularities that generalizes the class of smoothable singularities: these are all singularities that admit deformations to deficient conormal (dc) singularities. I will discuss how this new class arises from problems in differential equisingularity and how it relates to the vanishing of the local volume of a line bundle. I will discuss how the notion of dc singularities can be used to prove that normal rigid surfaces are smooth.

Steenrod's problem on realization of cycles. Old and new

Vasily Rozhdestvensky

Higher School of Economics

We will discuss the classical N. Steenrod's problem on realization of (integral singular) homology classes by continuous images of smooth manifolds. Apart from discussing classical results, I will present some of my own. Namely, the construction in each degree n of homology class x such that the minimal multiplicity with which it is realizable is equal to the maximal odd divisor of $[(n-1)/2]!$ and proving that in all dimensions less or equal to 23 this estimate is exact.

On the homology of Torelli groups

Igor Spiridonov

Higher School of Economics

Mapping class groups of oriented surfaces are closely related to moduli spaces of complex curves, and to topology of 3-manifolds. The "non-linear part" of the mapping class group is the Torelli subgroup $\mathcal{I}_g$, consisting of all mapping classes acting trivially on the first homology of the

surface. From a topological point of view, this subgroup is interesting because of its connection to homology 3-spheres. The Torelli group also arises in algebraic geometry as the fundamental group of the Torelli space, the moduli space of smooth complex curves with homology framings. The first homology group $\mathrm{H}_1(\mathcal{I}_g, \mathbb{Z})$ was described by Johnson in the 1980's. However, none of the other non-zero homology groups has been computed explicitly. In this talk, we give a complete description of the Torelli group top homology group in genus 3. The main approach is the study of the action of $\mathcal{I}_g$ on the complex of cycles, introduced by Bestvina, Bux and Margalit in 2007.

Quantization of symplectic varieties over the ring of integers

Vadim Vologodsky

Higher School of Economics

Given a smooth scheme X over a commutative ring containing $1/2$ with a locally exact symplectic form I will explain a construction of a canonical quantization of the category of quasi-coherent sheaves on X. This talk is based on a work by Roman Travkin assisted by Ekaterina Bogdanova, Dmitry Kubrak, and myself.

A Noether–Lefschetz Theorem for Spectral Varieties with Applications

Bin Wang

Steklov Mathematical Institute

We calculate the Picard group of generic (very general) spectral varieties living in the total space of a very ample line bundle over an algebraically closed field k of odd characteristics or characteristic 0. We follow the strategy of Ravindra and Srinivas via formal Picard groups. As an application, we calculate the generic fibers of Hitchin systems over a smooth quintic surface of Picard number 1. This is joint work with Xiaoyu Su.

Lagrangian SYZ fibrations

Ilia Zharkov

Kansas State University

I will show how to compactify a semi-simple SYZ fibration to a topological (orbi)-manifold. Then I will argue that one can choose this compactification to have a symplectic structure which extends the cotangent torus fibration from the smooth part of the SYZ base. Most of the time I will speak on a Liouville structure and its the Lagrangian skeleton of a local model $x_1 \dots x_k = 1 + w_1 + \dots + w_n$. This is a joint work in progress with Helge Ruddat.


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