Classification of degenerations and Picard lattices of Kahlerian K3 surfaces with finite symplectic automorphism groups.

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Abstract: I will speak about these my results which I obtained during last years – 2013—2020. This classification is almost finished now. Only for very small symplectic automorphism groups — of order 4, 3, 2 and 1 — it is not completely finished now.

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1 Classification of degenerations of K3 surfaces with finite symplectic automorphism groups.

Let X be a complex K3-surface. That is X is a compact complex surface which has a non-zero holomorphic differential form $\omega_X \in \Omega^2[X]$ with zero divisor ($K_X = 0$) and with the irregularity $q(X) = \dim \Omega^1[X] = 0$. Then $H^2(X,\mathbb{Z})$ with the intersection pairing is an even unimodular lattice L_{K3} with the signature $(3,19)$. For a non-zero holomorphic 2-form $\omega_X \in \Omega^2[X]$ we have $H^{2,0}(X) = \Omega^2[X] = \mathbb{C}\omega_X$. The primitive sublattice

$$S_X = H^2(X,\mathbb{Z}) \cap H^{1,1}(X) = \{x \in H^2(X,\mathbb{Z}) \mid x \cdot H^{2,0}(X) = 0\} \subset H^2(X,\mathbb{Z})$$

is *the Picard lattice* of X generated by 1-st Chern classes of all linear bundles over X. Primitive sublattice means that $H^2(X,\mathbb{Z})/S_X$ does not have a torsion.

Let G be a finite symplectic automorphism group of X. Symplectic means that for any $g \in G$, a non-zero holomorphic differential form $\omega_X \in H^{2,0}(X) = \Omega^2[X] = \mathbb{C}\omega_X$ is preserved: $g^*(\omega_X) = \omega_X$. For a G-invariant sublattice $M \subset H^2(X,\mathbb{Z})$, by $M^G = \{x \in M \mid G(x) = x\}$ we denote *the fixed sublattice of M*, and by $M_G = (M^G)^\perp_M$ we denote *the coinvariant sublattice of M.*

By [Nik1], [Nik2], the coinvariant sublattice $S_G = H^2(X,\mathbb{Z})_G = (S_X)_G$ is *a Leech type lattice:* S_G is negative definite, it has no elements with square (-2), G acts trivially on the discriminant group $A_{S_G} = (S_G)^*/S_G$, and $(S_G)^G = \{0\}$.

For a general pair (X,G), the Picard lattice $S_X = S_G$, and arbitrary (X,G) can be considered as Kählerian K3 surfaces with the condition $S_G \subset S_X$ on the Picard lattice (in terminology of [Nik2]). The dimension of their moduli is equal to $20 - \operatorname{rk} S_G$.

Let $E \subset X$ be a non-singular irreducible rational curve (that is $E \cong \mathbb{P}^1$). Equivalently, $\alpha = cl(E) \in S_X$ has $\alpha^2 = -2$, α is effective and α is numerically effective: $\alpha \cdot D \geq 0$ for any irreducible curve D of X such that $cl(D) \neq \alpha$.

Let us consider t non-singular irreducible rational curves $E_1, \dots, E_t$ of X with classes $\alpha_i = cl(E_i) \in S_X$ such that their orbits $G(E_1), \dots, G(E_t)$ are different (that is they don't intersect).
Let us consider the primitive sublattice

$$S = [S_G, G(\alpha_1), \dots, G(\alpha_t))]_{pr} = [S_G, \alpha_1, \dots, \alpha_t]_{pr} \subset S_X$$

of Picard lattice S_X which is generated by the coinvariant sublattice S_G and by all classes of orbits $G(E_1), \dots G(E_t)$, and we assume that *S is negative definite.*

Since S_G has no elements with square (-2), we have

$$\operatorname{rk} S = \operatorname{rk} S_G + t \le 19$$

(19 is the number of negative squares of the lattice $H^2(X,\mathbb{Z})$), and elements of orbits $G(\alpha_1), \dots G(\alpha_t)$ give the bases of the root system $\Delta(S)$ of elements with square (-2) in S.

All curves $G(E_1), \dots, G(E_t)$ of X can be contracted to Du Val singularities of the types of connected components of the Dynkin diagram of the basis (ADE-singularities). The group G will act on the corresponding singular K3 surface $\overline{X}$ with the Du Val singularities.

For a general such data $(X, G, G(E_1), \dots,\ G(E_t))$, the Picard lattice $S_X = S$ and the data can be considered as *a degeneration of the codimension* t of Kählerian K3 surfaces (X, G) with finite symplectic automorphism groups G. Really, the dimension of moduli of Kählerian K3 surfaces with the condition $S \subset S_X$ on the Picard lattice is equal to

$$20 - \operatorname{rk} S = 20 - \operatorname{rk} S_G - t = (20 - \operatorname{rk} S_G) - t,$$

where $20 - \operatorname{rk} S_G$ is the dimension of moduli of pairs (X, G).

We can consider only maximal finite symplectic automorphism groups G with the same coinvariant lattice S_G that is $G = Clos(G)$. By global Torelli Theorem for K3 surfaces [PS], [BR], it is equivalent to

$$G|S_G = \{g \in O(S_G) \mid g\ is\ identity\ on\ A_{S_G} = (S_G)^*/S_G\},$$

in particular, G is defined by the coinvariant lattice S_G.

The type of the degeneration is given by the Dynkin diagram and by its Dynkin subdiagrams

$$(Dyn(G(\alpha_1)), \dots, Dyn(G(\alpha_t))) \subset Dyn(G(\alpha_1) \cup \cdots \cup G(\alpha_t))$$

and their types. Numeration of Dynkin subdiagrams $Dyn(G(\alpha_i))$ and connected components of the Dynkin diagram $Dyn(G(\alpha_1) \cup \cdots \cup G(\alpha_t))$ should agree.

In difficult cases, we also consider the matrix of subdiagrams which is given by Dynkin subdiagrams

$$(Dyn(G(\alpha_i)),\ Dyn(G(\alpha_j))) \subset Dyn(G(\alpha_i) \cup G(\alpha_j))$$

and their types for $1 \le i < j \le t$.

By global Torelli Theorem for K3 surfaces [PS], [BR], **the type of the abstract group $G = Clos(G)$, which is equivalent to the isomorphism class of the coinvariant lattice S_G, and the type of the degeneration give the main invariants of the degeneration. The Picard lattice S gives the much more delicate invariant of the degeneration.**

For groups $G = Clos(G)$ of the order $|G| > 6$, the classification of the types of degenerations and the Picard lattices S is given in tables of [Nik9]—[Nik12].

I give them below and comment. They give the main results of the classification.

Table 1 gives classification of degenerations of the codim$= 1$*.* Of course, it is the most important because it gives a description of orbits $G(E_i)$. All of them have types $m\mathbb{A}_1$ or $m\mathbb{A}_2$.

a) We have $\mathrm{rk}\, S_G \leq 18$ because $\mathrm{rk}\, S_G + 1 \leq 19$. Groups G with $\mathrm{rk}\, S_G = 19$ have no degenerations, and they are not presented in Table 1. Lattices $S = S_G$ of all ranks were classified in my papers (for Abelian groups) 1979, Sh. Mukai 1988, Xiao 1996, Kondō 1998, Hashimoto 2010).

b) For degenerations $(\mathbf{n} = 10(G = D_8), 2\mathbb{A}_1)$ and $(\mathbf{n} = \mathbf{34}(G = \mathfrak{S}_4,), 6\mathbb{A}_1))$ only, the Picard lattice S of the degeneration is not defined by the type: for these cases, there are two possibilities for S.

All possibilities are given in Table 1.

Table 2 gives classification of degenerations of codimensions $t \geq 2$ *for groups* G *with* $\mathbf{n} \geq 12$*. Equivalently, either* $|G| \geq 12$ *or* $G = Q_8$*.*

Then $\mathrm{rk}\, S_G \leq 19 - t \leq 17$, and groups G with $\mathrm{rk}\, S_G = 19$, 18 are absent in this table.

Genuses of the Picard lattices S of the degenerations are given in the Table. By $*$, I denote cases (almost in all cases) when I prove that the lattice S is unique up to isomorphism.

Table 3 gives similar classification of degenerations of codimensions $t \geq 2$ for the remaining group with $\mathbf{n} = 10$*, that is* $G = D_8$. Then $\mathrm{rk}\, S_G = 15$, and there are degenerations of codimensions $t = 2,\ 3,\ 4$. There are many possibilities.

Table 4 gives similar classification of degenerations of codimensions $t \geq 2$ for the remaining group G with $\mathbf{n} = 9$*, that is* $G = (C_2)^3$. Then $\mathrm{rk}\, S_G = 14$, and there are degenerations of codimensions $t = 2,\ 3,\ 4,\ 5$. There are not so many possibilities than for the previous case.

Additionally to Tables 1—4, in Table 5, I give
List 1, which is important for the classification of the Picard lattices S for K3 surfaces. In tables 1—4, I additionally denote by o (old) cases when a degeneration of K3 surfaces with a symplectic automorphism group G_1 has, actually, larger symplectic automorphism group G_2 with

$$|G_1| < |G_2|.$$

The group G_2 has less orbits and less codimension than G_1. For classification of Picard lattices S of K3 surfaces, the lines of Tables 1—4, which are denoted by o, must be deleted. In **List 1**, the type of the degeneration for the group G_1 is shown to the left from the sign $\Longleftarrow$, and for the group G_2 it is shown to the right.

These cases are very interesting since they give cases when for a degeneration the finite symplectic automorphism group G_1 increases to G_2 surprisingly.

These cases are similar to the case which I had found in 1975, when I had shown that if a K3 gets 16 (sixteen) not intersected $\mathbb{P}^1$ (a degeneration $16\mathbb{A}_1$), then it is Kummer and it gets a symplectic automorphism group $(C_2)^4$ with the orbit $16\mathbb{A}_1$. That is C_1 increases to $(C_2)^4$.

Table 1: Types and Picard lattices S of degenerations of codimension 1 of Kählerian K3 surfaces with finite symplectic automorphism group $G = Clos(G)$.

n	$\|G\|$	i	G	$\text{rk}\, S_G$	q_{S_G}	Deg	$\text{rk}\, S$	q_S
1	2	1	C_2	8	2_{II}^{+8}	$\mathbb{A}_1$	9	2_7^{+9}
						$2\mathbb{A}_1$	9	$2_{II}^{-6}, 4_3^{-1}$
2	3	1	C_3	12	3^{+6}	$\mathbb{A}_1$	13	$2_3^{-1}, 3^{+6}$
						$3\mathbb{A}_1$	13	$2_1^{+1}, 3^{-5}$
3	4	2	C_2^2	12	$2_{II}^{-6}, 4_{II}^{-2}$	$\mathbb{A}_1$	13	$2_3^{+7}, 4_{II}^{+2}$
						$2\mathbb{A}_1$	13	$2_{II}^{-4}, 4_7^{-3}$
						$4\mathbb{A}_1$	13	$2_{II}^{-6}, 8_3^{-1}$
4	4	1	C_4	14	$2_2^{+2}, 4_{II}^{+4}$	$\mathbb{A}_1$	15	$2_5^{-3}, 4_{II}^{+4}$
						$2\mathbb{A}_1$	15	4_1^{-5}
						$4\mathbb{A}_1$	15	$2_2^{+2}, 4_{II}^{+2}, 8_7^{+1}$
						$\mathbb{A}_2$	15	$2_1^{+1}, 4_{II}^{-4}$
6	6	1	D_6	14	$2_{II}^{-2}, 3^{+5}$	$\mathbb{A}_1$	15	$2_7^{-3}, 3^{+5}$
						$2\mathbb{A}_1$	15	$4_3^{-1}, 3^{+5}$
						$3\mathbb{A}_1$	15	$2_1^{-3}, 3^{-4}$
						$6\mathbb{A}_1$	15	$4_1^{+1}, 3^{+4}$
9	8	5	C_2^3	14	$2_{II}^{+6}, 4_2^{+2}$	$2\mathbb{A}_1$	15	$2_{II}^{-4}, 4_5^{-3}$
						$4\mathbb{A}_1$	15	$2_{II}^{+6}, 8_1^{+1}$
						$8\mathbb{A}_1$	15	$2_{II}^{+6}, 4_1^{+1}$
10	8	3	D_8	15	4_1^{+5}	$\mathbb{A}_1$	16	$2_1^{+1}, 4_7^{+5}$
						$(2\mathbb{A}_1)_I$	16	$2_6^{-2}, 4_6^{-4}$
						$(2\mathbb{A}_1)_{II}$	16	$2_{II}^{+2}, 4_{II}^{+4}$
						$4\mathbb{A}_1$	16	$4_7^{+3}, 8_1^{+1}$
						$8\mathbb{A}_1$	16	4_0^{+4}
						$2\mathbb{A}_2$	16	4_{II}^{+4}
12	8	4	Q_8	17	$2_7^{-3}, 8_{II}^{-2}$	$8\mathbb{A}_1$	18	$2_7^{-3}, 16_3^{-1}$
						$\mathbb{A}_2$	18	$2_6^{-2}, 8_{II}^{-2}$
16	10	1	D_{10}	16	5^{+4}	$\mathbb{A}_1$	17	$2_7^{+1}, 5^{+4}$
						$5\mathbb{A}_1$	17	$2_7^{+1}, 5^{-3}$

n	$\|G\|$	i	G	rk S_G	q_{S_G}	Deg	rk S	q_S
17	12	3	$\mathfrak{A}_4$	16	$2_{II}^{-2}, 4_{II}^{-2}, 3^{+2}$	$\mathbb{A}_1$	17	$2_7^{-3}, 4_{II}^{+2}, 3^{+2}$
						$3\mathbb{A}_1$	17	$2_1^{-3}, 4_{II}^{+2}, 3^{-1}$
						$4\mathbb{A}_1$	17	$2_{II}^{-2}, 8_3^{-1}, 3^{+2}$
						$6\mathbb{A}_1$	17	$4_1^{-3}, 3^{+1}$
						$12\mathbb{A}_1$	17	$2_{II}^{-2}, 8_1^{+1}, 3^{-1}$
18	12	4	D_{12}	16	$2_{II}^{+4}, 3^{+4}$	$\mathbb{A}_1$	17	$2_7^{+5}, 3^{+4}$
						$2\mathbb{A}_1$	17	$2_{II}^{+2}, 4_7^{+1}, 3^{+4}$
						$3\mathbb{A}_1$	17	$2_5^{-5}, 3^{-3}$
						$6\mathbb{A}_1$	17	$2_{II}^{-2}, 4_1^{+1}, 3^{+3}$
21	16	14	C_2^4	15	$2_{II}^{+6}, 8_I^{+1}$	$4\mathbb{A}_1$	16	$2_{II}^{+4}, 4_{II}^{+2}$
						$16\mathbb{A}_1$	16	2_{II}^{+6}
22	16	11	$C_2 \times D_8$	16	$2_{II}^{+2}, 4_0^{+4}$	$2\mathbb{A}_1$	17	4_7^{+5}
						$4\mathbb{A}_1$	17	$2_{II}^{+2}, 4_0^{+2}, 8_7^{+1}$
						$8\mathbb{A}_1$	17	$2_{II}^{+2}, 4_7^{+3}$
26	16	8	SD_{16}	18	$2_7^{+1}, 4_7^{+1}, 8_{II}^{+2}$	$8\mathbb{A}_1$	19	$2_1^{+1}, 4_1^{+1}, 16_3^{-1}$
						$2\mathbb{A}_2$	19	$2_5^{-1}, 8_{II}^{-2}$
30	18	4	$\mathfrak{A}_{3,3}$	16	$3^{+4}, 9^{-1}$	$3\mathbb{A}_1$	17	$2_5^{-1}, 3^{-3}, 9^{-1}$
						$9\mathbb{A}_1$	17	$2_3^{-1}, 3^{+4}$
32	20	3	$Hol(C_5)$	18	$2_6^{-2}, 5^{+3}$	$2\mathbb{A}_1$	19	$4_1^{+1}, 5^{+3}$
						$5\mathbb{A}_1$	19	$2_1^{+3}, 5^{-2}$
						$10\mathbb{A}_1$	19	$4_5^{-1}, 5^{+2}$
						$5\mathbb{A}_2$	19	$2_5^{-1}, 5^{-2}$
33	21	1	$C_7 \rtimes C_3$	18	7^{+3}	$7\mathbb{A}_1$	19	$2_1^{+1}, 7^{+2}$
34	24	12	$\mathfrak{S}_4$	17	$4_3^{+3}, 3^{+2}$	$\mathbb{A}_1$	18	$2_5^{-1}, 4_1^{+3}, 3^{+2}$
						$2\mathbb{A}_1$	18	$2_2^{+2}, 4_{II}^{+2}, 3^{+2}$
						$3\mathbb{A}_1$	18	$2_7^{+1}, 4_5^{-3}, 3^{-1}$
						$4\mathbb{A}_1$	18	$4_3^{-1}, 8_3^{-1}, 3^{+2}$
						$(6\mathbb{A}_1)_I$	18	$2_4^{-2}, 4_0^{+2}, 3^{+1}$
						$(6\mathbb{A}_1)_{II}$	18	$2_{II}^{+2}, 4_{II}^{-2}, 3^{+1}$
						$8\mathbb{A}_1$	18	$4_2^{+2}, 3^{+2}$
						$12\mathbb{A}_1$	18	$4_5^{-1}, 8_7^{+1}, 3^{-1}$
						$6\mathbb{A}_2$	18	$4_{II}^{-2}, 3^{+1}$

n	$\lvert G\rvert$	i	G	rk S_G	q_{S_G}	Deg	rk S	q_S
39	32	27	2^4C_2	17	$2_{II}^{+2}, 4_0^{+2}, 8_7^{+1}$	$4\mathbb{A}_1$	18	4_6^{+4}
						$8\mathbb{A}_1$	18	$2_{II}^{+2}, 4_7^{+1}, 8_7^{+1}$
						$16\mathbb{A}_1$	18	$2_{II}^{+2}, 4_6^{+2}$
40	32	49	$Q_8 * Q_8$	17	4_7^{+5}	$8\mathbb{A}_1$	18	4_6^{+4}
46	36	9	3^2C_4	18	$2_6^{-2}, 3^{+2}, 9^{-1}$	$6\mathbb{A}_1$	19	$4_7^{+1}, 3^{+1}, 9^{-1}$
						$9\mathbb{A}_1$	19	$2_5^{-3}, 3^{+2}$
						$9\mathbb{A}_2$	19	$2_5^{-1}, 3^{+2}$
48	36	10	$\mathfrak{S}_{3,3}$	18	$2_{II}^{-2}, 3^{+3}, 9^{-1}$	$3\mathbb{A}_1$	19	$2_5^{+3}, 3^{-2}, 9^{-1}$
						$6\mathbb{A}_1$	19	$4_1^{+1}, 3^{+2}, 9^{-1}$
						$9\mathbb{A}_1$	19	$2_7^{-3}, 3^{+3}$
49	48	50	2^4C_3	17	$2_{II}^{-4}, 8_1^{+1}, 3^{-1}$	$4\mathbb{A}_1$	18	$2_{II}^{-2}, 4_{II}^{+2}, 3^{-1}$
						$12\mathbb{A}_1$	18	$2_{II}^{-2}, 4_2^{-2}$
						$16\mathbb{A}_1$	18	$2_{II}^{-4}, 3^{-1}$
51	48	48	$C_2 \times \mathfrak{S}_4$	18	$2_{II}^{+2}, 4_2^{+2}, 3^{+2}$	$2\mathbb{A}_1$	19	$4_1^{+3}, 3^{+2}$
						$4\mathbb{A}_1$	19	$2_{II}^{+2}, 8_1^{+1}, 3^{+2}$
						$6\mathbb{A}_1$	19	$4_7^{-3}, 3^{+1}$
						$8\mathbb{A}_1$	19	$2_{II}^{-2}, 4_5^{-1}, 3^{+2}$
						$12\mathbb{A}_1$	19	$2_{II}^{-2}, 8_7^{+1}, 3^{-1}$
55	60	5	$\mathfrak{A}_5$	18	$2_{II}^{-2}, 3^{+1}, 5^{-2}$	$\mathbb{A}_1$	19	$2_7^{-3}, 3^{+1}, 5^{-2}$
						$5\mathbb{A}_1$	19	$2_3^{+3}, 3^{+1}, 5^{+1}$
						$6\mathbb{A}_1$	19	$4_1^{+1}, 5^{-2}$
						$10\mathbb{A}_1$	19	$4_7^{+1}, 3^{+1}, 5^{-1}$
						$15\mathbb{A}_1$	19	$2_5^{+3}, 5^{-1}$
56	64	138	$\Gamma_{25}a_1$	18	$4_5^{+3}, 8_1^{+1}$	$8\mathbb{A}_1$	19	$4_4^{-2}, 8_5^{-1}$
						$16\mathbb{A}_1$	19	4_5^{+3}
61	72	43	$\mathfrak{A}_{4,3}$	18	$4_{II}^{-2}, 3^{-3}$	$3\mathbb{A}_1$	19	$2_5^{-1}, 4_{II}^{+2}, 3^{+2}$
						$12\mathbb{A}_1$	19	$8_1^{+1}, 3^{+2}$
65	96	227	2^4D_6	18	$2_{II}^{-2}, 4_7^{+1}, 8_1^{+1}, 3^{-1}$	$4\mathbb{A}_1$	19	$4_3^{-3}, 3^{-1}$
						$8\mathbb{A}_1$	19	$2_{II}^{-2}, 8_7^{+1}, 3^{-1}$
						$12\mathbb{A}_1$	19	4_5^{+3}
						$16\mathbb{A}_1$	19	$2_{II}^{+2}, 4_3^{-1}, 3^{-1}$
75	192	1023	$4^2\mathfrak{A}_4$	18	$2_{II}^{-2}, 8_6^{-2}$	$16\mathbb{A}_1$	19	$2_{II}^{-2}, 8_5^{-1}$

Table 2: Types and lattices S of degenerations of codimension ≥ 2 of Kählerian K3 surfaces with finite symplectic automorphism groups $G = Clos(G)$ for $\mathbf{n} \geq 12$.

$\mathbf{n}$	$\lvert G\rvert$	i	G	$\mathrm{rk}\, S_G$	Deg	$\mathrm{rk}\, S$	q_S
12	8	4	Q_8	17	$(8\mathbb{A}_1, 8\mathbb{A}_1) \subset 16\mathbb{A}_1$ o	19	$2_{II}^{-2}, 8_5^{-1}$ $*$
					$(8\mathbb{A}_1, 8\mathbb{A}_1) \subset 8\mathbb{A}_2$	19	$2_7^{-3}, 3^{-1}$ $*$
					$(8\mathbb{A}_1, \mathbb{A}_2) \subset 8\mathbb{A}_1 \amalg \mathbb{A}_2$	19	$2_2^{+2}, 16_3^{-1}$ $*$
					$(\mathbb{A}_2, \mathbb{A}_2) \subset 2\mathbb{A}_2$ o	19	$2_5^{-1}, 8_{II}^{-2}$ $*$
16	10	1	D_{10}	16	$(\mathbb{A}_1, \mathbb{A}_1) \subset 2\mathbb{A}_1$	18	$2_6^{+2}, 5^{+4}$
					$(\mathbb{A}_1, 5\mathbb{A}_1) \subset 6\mathbb{A}_1$	18	$2_6^{+2}, 5^{-3}$
					$(5\mathbb{A}_1, 5\mathbb{A}_1) \subset 10\mathbb{A}_1$	18	$2_6^{+2}, 5^{+2}$ $*$
					$(5\mathbb{A}_1, 5\mathbb{A}_1) \subset 5\mathbb{A}_2$	18	$3^{-1}, 5^{-2}$ $*$
					$(\mathbb{A}_1, \mathbb{A}_1, 5\mathbb{A}_1) \subset 7\mathbb{A}_1$	19	$2_5^{+3}, 5^{-3}$
					$(\mathbb{A}_1, 5\mathbb{A}_1, 5\mathbb{A}_1) \subset 11\mathbb{A}_1$	19	$2_5^{+3}, 5^{+2}$
					$(\mathbb{A}_1, 5\mathbb{A}_1, 5\mathbb{A}_1) \subset \mathbb{A}_1 \amalg 5\mathbb{A}_2$	19	$2_7^{+1}, 3^{-1}, 5^{-2}$
					$(5\mathbb{A}_1, 5\mathbb{A}_1, 5\mathbb{A}_1) \subset 15\mathbb{A}_1$	19	$2_1^{-3}, 5^{-1}$ $*$
					$(5\mathbb{A}_1, 5\mathbb{A}_1, 5\mathbb{A}_1) \subset 5\mathbb{A}_2 \amalg 5\mathbb{A}_1$	19	$2_7^{+1}, 3^{-1}, 5^{+1}$ $*$
					$\begin{pmatrix} 5\mathbb{A}_1 & 5\mathbb{A}_2 & 10\mathbb{A}_1 \\ & 5\mathbb{A}_1 & 5\mathbb{A}_2 \\ & & 5\mathbb{A}_1 \end{pmatrix} \subset 5\mathbb{A}_3$	19	$4_1^{+1}, 5^{+1}$ $*$
17	12	3	$\mathfrak{A}_4$	16	$(\mathbb{A}_1, \mathbb{A}_1) \subset 2\mathbb{A}_1$ o	18	$2_2^{+2}, 4_{II}^{+2}, 3^{+2}$ $*$
					$(\mathbb{A}_1, 3\mathbb{A}_1) \subset 4\mathbb{A}_1$ o	18	$2_{II}^{-2}, 4_{II}^{+2}, 3^{-1}$ $*$
					$(\mathbb{A}_1, 4\mathbb{A}_1) \subset 5\mathbb{A}_1$	18	$2_3^{+3}, 8_7^{+1}, 3^{+2}$ $*$
					$(\mathbb{A}_1, 6\mathbb{A}_1) \subset 7\mathbb{A}_1$	18	$2_1^{+1}, 4_7^{+3}, 3^{+1}$ $*$
					$(\mathbb{A}_1, 12\mathbb{A}_1) \subset 13\mathbb{A}_1$	18	$2_3^{+3}, 8_1^{+1}, 3^{-1}$ $*$
					$(3\mathbb{A}_1, 4\mathbb{A}_1) \subset 7\mathbb{A}_1$	18	$2_5^{+3}, 8_7^{+1}, 3^{-1}$ $*$
					$(3\mathbb{A}_1, 6\mathbb{A}_1) \subset 3\mathbb{A}_3$	18	$2_6^{+2}, 4_{II}^{-2}$ $*$
					$(3\mathbb{A}_1, 12\mathbb{A}_1) \subset 15\mathbb{A}_1$	18	$2_1^{-3}, 8_1^{+1}$ $*$
					$(4\mathbb{A}_1, 4\mathbb{A}_1) \subset 8\mathbb{A}_1$ o	18	$4_2^{+2}, 3^{+2}$ $*$
					$(4\mathbb{A}_1, 4\mathbb{A}_1) \subset 4\mathbb{A}_2$	18	$2_{II}^{-2}, 3^{+3}$ $*$
					$(4\mathbb{A}_1, 6\mathbb{A}_1) \subset 10\mathbb{A}_1$	18	$4_1^{+1}, 8_3^{-1}, 3^{+1}$ $*$
					$(4\mathbb{A}_1, 12\mathbb{A}_1) \subset 16\mathbb{A}_1$ o	18	$2_{II}^{-4}, 3^{-1}$ $*$
					$(4\mathbb{A}_1, 12\mathbb{A}_1) \subset 4\mathbb{D}_4$	18	$2_{II}^{-2}, 3^{-1}$ $*$
					$(6\mathbb{A}_1, 6\mathbb{A}_1) \subset 12\mathbb{A}_1$ o	18	$2_{II}^{-2}, 4_2^{-2}$ $*$

					$(6\mathbb{A}_1, 6\mathbb{A}_1) \subset 6\mathbb{A}_2\ o$	18	$4_{II}^{-2}, 3^{+1}\ *$
					$(6\mathbb{A}_1, 12\mathbb{A}_1) \subset 6\mathbb{A}_3$	18	$4_6^{+2}\ *$
					$(\mathbb{A}_1, \mathbb{A}_1, \mathbb{A}_1) \subset 3\mathbb{A}_1\ o$	19	$2_1^{+1}, 4_{II}^{+2}, 3^{+2}\ *$
					$(\mathbb{A}_1, \mathbb{A}_1, 4\mathbb{A}_1) \subset 6\mathbb{A}_1\ o$	19	$2_2^{+2}, 8_7^{+1}, 3^{+2}\ *$
					$(\mathbb{A}_1, \mathbb{A}_1, 6\mathbb{A}_1) \subset 8\mathbb{A}_1\ o$	19	$4_7^{-3}, 3^{+1}\ *$
					$(\mathbb{A}_1, \mathbb{A}_1, 12\mathbb{A}_1) \subset 14\mathbb{A}_1\ \ o$	19	$2_2^{+2}, 8_5^{-1}, 3^{-1}\ *$
					$(\mathbb{A}_1, 3\mathbb{A}_1, 4\mathbb{A}_1) \subset 8\mathbb{A}_1\ o$	19	$2_{II}^{-2}, 8_7^{+1}, 3^{-1}\ *$
					$(\mathbb{A}_1, 3\mathbb{A}_1, 12\mathbb{A}_1) \subset 16\mathbb{A}_1\ o$	19	$2_{II}^{-2}, 8_5^{-1}\ *$
					$(\mathbb{A}_1, 4\mathbb{A}_1, 4\mathbb{A}_1) \subset 9\mathbb{A}_1\ o$	19	$2_1^{+1}, 4_0^{+2}, 3^{+2}\ *$
					$(\mathbb{A}_1, 4\mathbb{A}_1, 4\mathbb{A}_1) \subset \mathbb{A}_1 \amalg 4\mathbb{A}_2$	19	$2_7^{-3}, 3^{+3}\ *$
					$(\mathbb{A}_1, 4\mathbb{A}_1, 6\mathbb{A}_1) \subset 11\mathbb{A}_1$	19	$2_3^{-1}, 4_7^{+1}, 8_5^{-1}, 3^{+1}$
					$(\mathbb{A}_1, 4\mathbb{A}_1, 12\mathbb{A}_1) \subset \mathbb{A}_1 \amalg 4\mathbb{D}_4$	19	$2_7^{-3}, 3^{-1}\ *$
					$(\mathbb{A}_1, 6\mathbb{A}_1, 6\mathbb{A}_1) \subset \mathbb{A}_1 \amalg 6\mathbb{A}_2\ o$	19	$2_3^{-1}, 4_{II}^{-2}, 3^{+1}\ *$
					$(\mathbb{A}_1, 6\mathbb{A}_1, 12\mathbb{A}_1) \subset \mathbb{A}_1 \amalg 6\mathbb{A}_3$	19	$2_7^{+1}, 4_6^{+2}$
					$(3\mathbb{A}_1, 4\mathbb{A}_1, 4\mathbb{A}_1) \subset 11\mathbb{A}_1\ \ o$	19	$2_1^{+1}, 4_2^{+2}, 3^{-1}\ *$
					$(3\mathbb{A}_1, 4\mathbb{A}_1, 4\mathbb{A}_1) \subset 3\mathbb{A}_1 \amalg 4\mathbb{A}_2$	19	$2_5^{+3}, 3^{-2}\ *$
					$(3\mathbb{A}_1, 4\mathbb{A}_1, 12\mathbb{A}_1) \subset 3\mathbb{A}_1 \amalg 4\mathbb{D}_4$	19	$2_5^{+3}\ *$
					$(3\mathbb{A}_1, 6\mathbb{A}_1, 4\mathbb{A}_1) \subset 3\mathbb{A}_3 \amalg 4\mathbb{A}_1$	19	$2_6^{+2}, 8_3^{-1}$
					$(4\mathbb{A}_1, 4\mathbb{A}_1, 4\mathbb{A}_1) \subset 12\mathbb{A}_1$	19	$8_1^{+1}, 3^{+2}\ *$
					$\begin{pmatrix} 4\mathbb{A}_1 & 4\mathbb{A}_2 & 8\mathbb{A}_1 \\ & 4\mathbb{A}_1 & 4\mathbb{A}_2 \\ & & 4\mathbb{A}_1 \end{pmatrix} \subset 4\mathbb{A}_3\ o$	19	$4_1^{+1}, 3^{+2}\ *$
					$(4\mathbb{A}_1, 4\mathbb{A}_1, 6\mathbb{A}_1) \subset 14\mathbb{A}_1\ \ o$	19	$2_2^{-2}, 4_1^{+1}, 3^{+1}\ *$
					$(4\mathbb{A}_1, 4\mathbb{A}_1, 6\mathbb{A}_1) \subset 4\mathbb{A}_2 \amalg 6\mathbb{A}_1$	19	$4_1^{+1}, 3^{+2}\ *$
					$(4\mathbb{A}_1, 6\mathbb{A}_1, 6\mathbb{A}_1) \subset 16\mathbb{A}_1\ o$	19	$2_{II}^{-2}, 8_5^{-1}\ *$
					$(4\mathbb{A}_1, 6\mathbb{A}_1, 6\mathbb{A}_1) \subset 4\mathbb{A}_1 \amalg 6\mathbb{A}_2\ \ o$	19	$8_3^{-1}, 3^{+1}\ *$

n	$\|G\|$	i	G	Deg	rk S	q_S
18	12	4	D_{12}	$(\mathbb{A}_1, 2\mathbb{A}_1) \subset 3\mathbb{A}_1$	18	$2_7^{+3}, 4_7^{+1}, 3^{+4}$
				$(\mathbb{A}_1, 3\mathbb{A}_1) \subset 4\mathbb{A}_1$	18	$2_0^{+4}, 3^{-3}$ *
				$(\mathbb{A}_1, 6\mathbb{A}_1) \subset 7\mathbb{A}_1$	18	$2_1^{-3}, 4_7^{+1}, 3^{+3}$ *
				$(2\mathbb{A}_1, 3\mathbb{A}_1) \subset 5\mathbb{A}_1$	18	$2_1^{+3}, 4_7^{+1}, 3^{-3}$
				$((2\mathbb{A}_1, 6\mathbb{A}_1) \subset 8\mathbb{A}_1)_I$	18	$4_4^{-2}, 3^{+3}$ *
				$((2\mathbb{A}_1, 6\mathbb{A}_1) \subset 8\mathbb{A}_1)_{II}$	18	$2_{II}^{-4}, 3^{+3}$
				$(2\mathbb{A}_1, 6\mathbb{A}_1) \subset 2\mathbb{D}_4$	18	$2_{II}^{-2}, 3^{+3}$ *
				$(3\mathbb{A}_1, 6\mathbb{A}_1) \subset 9\mathbb{A}_1$	18	$2_7^{+3}, 4_7^{+1}, 3^{-2}$ *
				$(3\mathbb{A}_1, 6\mathbb{A}_1) \subset 3\mathbb{A}_3$	18	$2_6^{-4}, 3^{+2}$ *
				$(6\mathbb{A}_1, 6\mathbb{A}_1) \subset 12\mathbb{A}_1$	18	$4_2^{+2}, 3^{+2}$ *
				$(6\mathbb{A}_1, 6\mathbb{A}_1) \subset 6\mathbb{A}_2$	18	$2_{II}^{-2}, 3^{-1}, 9^{-1}$ *
				$(\mathbb{A}_1, 2\mathbb{A}_1, 3\mathbb{A}_1) \subset 6\mathbb{A}_1$	19	$2_0^{+2}, 4_7^{+1}, 3^{-3}$ *
				$\begin{pmatrix} \mathbb{A}_1 & 3\mathbb{A}_1 & 7\mathbb{A}_1 \\ & 2\mathbb{A}_1 & (8\mathbb{A}_1)_I \\ & & 6\mathbb{A}_1 \end{pmatrix} \subset 9\mathbb{A}_1$	19	$2_1^{+1}, 4_2^{+2}, 3^{+3}$
				$\begin{pmatrix} \mathbb{A}_1 & 3\mathbb{A}_1 & 7\mathbb{A}_1 \\ & 2\mathbb{A}_1 & 2\mathbb{D}_4 \\ & & 6\mathbb{A}_1 \end{pmatrix} \subset \mathbb{A}_1 \amalg 2\mathbb{D}_4$	19	$2_7^{-3}, 3^{+3}$ *
				$(\mathbb{A}_1, 3\mathbb{A}_1, 6\mathbb{A}_1) \subset 10\mathbb{A}_1$	19	$2_2^{-2}, 4_7^{+1}, 3^{-2}$ *
				$\begin{pmatrix} \mathbb{A}_1 & 4\mathbb{A}_1 & 7\mathbb{A}_1 \\ & 3\mathbb{A}_1 & 3\mathbb{A}_3 \\ & & 6\mathbb{A}_1 \end{pmatrix} \subset \mathbb{A}_1 \amalg 3\mathbb{A}_3$	19	$2_1^{+3}, 3^{+2}$ *
				$(\mathbb{A}_1, 6\mathbb{A}_1, 6\mathbb{A}_1) \subset 13\mathbb{A}_1$	19	$2_7^{+1}, 4_2^{+2}, 3^{+2}$ *
				$(\mathbb{A}_1, 6\mathbb{A}_1, 6\mathbb{A}_1) \subset \mathbb{A}_1 \amalg 6\mathbb{A}_2$	19	$2_3^{+3}, 3^{-1}, 9^{-1}$ *
				$\begin{pmatrix} 2\mathbb{A}_1 & 5\mathbb{A}_1 & (8\mathbb{A}_1)_I \\ & 3\mathbb{A}_1 & 9\mathbb{A}_1 \\ & & 6\mathbb{A}_1 \end{pmatrix} \subset 11\mathbb{A}_1$	19	$2_7^{+1}, 4_6^{+2}, 3^{-2}$
				$\begin{pmatrix} 2\mathbb{A}_1 & 5\mathbb{A}_1 & (8\mathbb{A}_1)_I \\ & 3\mathbb{A}_1 & 3\mathbb{A}_3 \\ & & 6\mathbb{A}_1 \end{pmatrix} \subset 2\mathbb{A}_1 \amalg 3\mathbb{A}_3$	19	$2_2^{+2}, 4_7^{+1}, 3^{+2}$ *

				$\begin{pmatrix} 2\mathbb{A}_1 & 5\mathbb{A}_1 & (8\mathbb{A}_1)_{II} \\ & 3\mathbb{A}_1 & 3\mathbb{A}_3 \\ & & 6\mathbb{A}_1 \end{pmatrix} \subset 2\mathbb{A}_1 \amalg 3\mathbb{A}_3$	19	$2_{II}^{+2}, 4_1^{+1}, 3^{+2}$
				$\begin{pmatrix} 2\mathbb{A}_1 & 5\mathbb{A}_1 & 2\mathbb{D}_4 \\ & 3\mathbb{A}_1 & 9\mathbb{A}_1 \\ & & 6\mathbb{A}_1 \end{pmatrix} \subset 3\mathbb{A}_1 \amalg 2\mathbb{D}_4$	19	$2_5^{+3}, 3^{-2}$ *
				$\begin{pmatrix} 2\mathbb{A}_1 & (8\mathbb{A}_1)_I & (8\mathbb{A}_1)_{II} \\ & 6\mathbb{A}_1 & 12\mathbb{A}_1 \\ & & 6\mathbb{A}_1 \end{pmatrix} \subset 14\mathbb{A}_1$	19	$2_{II}^{+2}, 4_1^{+1}, 3^{+2}$
				$\begin{pmatrix} 2\mathbb{A}_1 & (8\mathbb{A}_1)_I & (8\mathbb{A}_1)_I \\ & 6\mathbb{A}_1 & 6\mathbb{A}_2 \\ & & 6\mathbb{A}_1 \end{pmatrix} \subset 2\mathbb{A}_1 \amalg 6\mathbb{A}_2$	19	$4_3^{-1}, 3^{-1}, 9^{-1}$ *
				$\begin{pmatrix} 2\mathbb{A}_1 & 2\mathbb{D}_4 & (8\mathbb{A}_1)_I \\ & 6\mathbb{A}_1 & 12\mathbb{A}_1 \\ & & 6\mathbb{A}_1 \end{pmatrix} \subset 2\mathbb{D}_4 \amalg 6\mathbb{A}_1$	19	$4_1^{+1}, 3^{+2}$ *
				$(3\mathbb{A}_1, 6\mathbb{A}_1, 6\mathbb{A}_1) \subset 15\mathbb{A}_1$	19	$2_3^{-1}, 4_4^{-2}, 3^{-1}$ *
				$(3\mathbb{A}_1, 6\mathbb{A}_1, 6\mathbb{A}_1) \subset 3\mathbb{A}_1 \amalg 6\mathbb{A}_2$	19	$2_1^{-3}, 9^{-1}$ *
				$\begin{pmatrix} 3\mathbb{A}_1 & 3\mathbb{A}_3 & 9\mathbb{A}_1 \\ & 6\mathbb{A}_1 & 12\mathbb{A}_1 \\ & & 6\mathbb{A}_1 \end{pmatrix} \subset 3\mathbb{A}_3 \amalg 6\mathbb{A}_1$	19	$2_6^{-2}, 4_5^{-1}, 3^{+1}$ *
				$\begin{pmatrix} 3\mathbb{A}_1 & 3\mathbb{A}_3 & 9\mathbb{A}_1 \\ & 6\mathbb{A}_1 & 6\mathbb{A}_2 \\ & & 6\mathbb{A}_1 \end{pmatrix} \subset 3\mathbb{A}_5$	19	$2_3^{+3}, 3^{-1}$ *
n	$\|G\|$	i	G	*Deg*	rk S	q_S
21	16	14	C_2^4	$(4\mathbb{A}_1, 4\mathbb{A}_1) \subset 8\mathbb{A}_1$	17	$2_{II}^{+4}, 8_7^{+1}$ *
				$(4\mathbb{A}_1, 4\mathbb{A}_1, 4\mathbb{A}_1) \subset 12\mathbb{A}_1$ *o*	18	$2_{II}^{-2}, 4_2^{-2}$ *
				$(4\mathbb{A}_1, 4\mathbb{A}_1, 4\mathbb{A}_1, 4\mathbb{A}_1) \subset 16\mathbb{A}_1$ *o*	19	$2_{II}^{-2}, 8_5^{-1}$ *

n	$\|G\|$	i	G	Deg	rk S	q_S
22	16	11	$C_2\times D_8$	$(2\mathbb{A}_1,2\mathbb{A}_1)\subset 4\mathbb{A}_1$ o	18	4_6^{+4} $*$
				$(2\mathbb{A}_1,4\mathbb{A}_1)\subset 6\mathbb{A}_1$	18	$4_7^{+3},8_7^{+1}$ $*$
				$(2\mathbb{A}_1,8\mathbb{A}_1)\subset 10\mathbb{A}_1$	18	4_6^{+4}
				$((4\mathbb{A}_1,4\mathbb{A}_1)\subset 8\mathbb{A}_1)_I$ o	18	4_6^{+4} $*$
				$((4\mathbb{A}_1,4\mathbb{A}_1)\subset 8\mathbb{A}_1)_{II}$	18	$2_{II}^{-2},8_6^{-2}$ $*$
				$(4\mathbb{A}_1,8\mathbb{A}_1)\subset 12\mathbb{A}_1$	18	$2_{II}^{+2},4_7^{+1},8_7^{+1}$ $*$
				$(4\mathbb{A}_1,8\mathbb{A}_1)\subset 4\mathbb{A}_3$	18	$2_{II}^{-2},4_2^{-2}$ $*$
				$(8\mathbb{A}_1,8\mathbb{A}_1)\subset 16\mathbb{A}_1$ o	18	$2_{II}^{+2},4_6^{+2}$ $*$
				$(2\mathbb{A}_1,2\mathbb{A}_1,4\mathbb{A}_1)\subset 8\mathbb{A}_1$ o	19	$4_6^{+2},8_7^{+1}$ $*$
				$(2\mathbb{A}_1,2\mathbb{A}_1,8\mathbb{A}_1)\subset 12\mathbb{A}_1$ o	19	4_5^{+3} $*$
				$(2\mathbb{A}_1,(4\mathbb{A}_1,4\mathbb{A}_1)_{II})\subset 10\mathbb{A}_1$	19	$4_5^{-1},8_4^{-2}$ $*$
				$(2\mathbb{A}_1,4\mathbb{A}_1,8\mathbb{A}_1)\subset 14\mathbb{A}_1$	19	$4_4^{-2},8_5^{-1}$
				$\begin{pmatrix} 2\mathbb{A}_1 & 6\mathbb{A}_1 & 10\mathbb{A}_1 \\ & 4\mathbb{A}_1 & 4\mathbb{A}_3 \\ & & 8\mathbb{A}_1 \end{pmatrix}\subset 2\mathbb{A}_1 \amalg 4\mathbb{A}_3$	19	4_5^{+3} $*$
				$((4\mathbb{A}_1,\ 4\mathbb{A}_1)_I,\ 8\mathbb{A}_1)\subset 16\mathbb{A}_1$ o	19	4_5^{+3} $*$
				$\begin{pmatrix} 4\mathbb{A}_1 & (8\mathbb{A}_1)_{II} & 12\mathbb{A}_1 \\ & 4\mathbb{A}_1 & 4\mathbb{A}_3 \\ & & 8\mathbb{A}_1 \end{pmatrix}\subset 4\mathbb{A}_1 \amalg 4\mathbb{A}_3$	19	$2_{II}^{-2},8_5^{-1}$ $*$
30	18	4	$\mathfrak{A}_{3,3}$	$(3\mathbb{A}_1,\ 3\mathbb{A}_1)\subset 6\mathbb{A}_1$	18	$2_2^{+2},3^{+2},9^{-1}$ $*$
				$(3\mathbb{A}_1,\ 9\mathbb{A}_1)\subset 12\mathbb{A}_1$	18	$2_0^{+3},3^{-3}$ $*$
				$(3\mathbb{A}_1,\ 9\mathbb{A}_1)\subset 3\mathbb{D}_4$	18	3^{-3} $*$
				$(9\mathbb{A}_1,\ 9\mathbb{A}_1)\subset 9\mathbb{A}_2$	18	3^{-3} $*$
				$(3\mathbb{A}_1,\ 3\mathbb{A}_1,\ 3\mathbb{A}_1)\subset 9\mathbb{A}_1$	19	$2_7^{-3},3^{-1},9^{-1}$ $*$
				$(3\mathbb{A}_1,\ 3\mathbb{A}_1,\ 9\mathbb{A}_1)\subset 15\mathbb{A}_1$	19	$2_1^{+3},3^{+2}$ $*$
				$\begin{pmatrix} 3\mathbb{A}_1 & 6\mathbb{A}_1 & 12\mathbb{A}_1 \\ & 3\mathbb{A}_1 & 3\mathbb{D}_4 \\ & & 9\mathbb{A}_1 \end{pmatrix}\subset 3\mathbb{A}_1 \amalg 3\mathbb{D}_4$	19	$2_1^{+1},3^{+2}$ $*$

n	$\|G\|$	i	G	rk S_G	Deg	rk S	q_S
34	24	12	$\mathfrak{S}_4$	17	$(\mathbb{A}_1, \mathbb{A}_1) \subset 2\mathbb{A}_1$ o	19	$4_1^{+3}, 3^{+2}$ $*$
					$(\mathbb{A}_1, 2\mathbb{A}_1) \subset 3\mathbb{A}_1$ o	19	$2_1^{+1}, 4_{II}^{+2}, 3^{+2}$ $*$
					$(\mathbb{A}_1, 3\mathbb{A}_1) \subset 4\mathbb{A}_1$ o	19	$4_3^{-3}, 3^{-1}$ $*$
					$(\mathbb{A}_1, 4\mathbb{A}_1) \subset 5\mathbb{A}_1$	19	$2_1^{+1}, 4_1^{+1}, 8_7^{+1}, 3^{+2}$ $*$
					$(\mathbb{A}_1, (6\mathbb{A}_1)_I) \subset 7\mathbb{A}_1$	19	$2_1^{+1}, 4_6^{-2}, 3^{+1}$ $*$
					$(\mathbb{A}_1, 8\mathbb{A}_1) \subset 9\mathbb{A}_1$	19	$2_1^{+1}, 4_0^{+2}, 3^{+2}$ $*$
					$(\mathbb{A}_1, 12\mathbb{A}_1) \subset 13\mathbb{A}_1$	19	$2_7^{+1}, 4_5^{-1}, 8_7^{+1}, 3^{-1}$
					$(\mathbb{A}_1, 6\mathbb{A}_2) \subset \mathbb{A}_1 \amalg 6\mathbb{A}_2$	19	$2_7^{+1}, 4_{II}^{+2}, 3^{+1}$ $*$
					$(2\mathbb{A}_1, 4\mathbb{A}_1) \subset 6\mathbb{A}_1$	19	$2_2^{+2}, 8_7^{+1}, 3^{+2}$ $*$
					$(2\mathbb{A}_1, (6\mathbb{A}_1)_{II}) \subset 8\mathbb{A}_1$	19	$4_7^{-3}, 3^{+1}$ $*$
					$(2\mathbb{A}_1, 12\mathbb{A}_1) \subset 14\mathbb{A}_1$	19	$2_2^{+2}, 8_1^{+1}, 3^{-1}$ $*$
					$(3\mathbb{A}_1, 4\mathbb{A}_1) \subset 7\mathbb{A}_1$	19	$2_1^{+1}, 4_1^{+1}, 8_5^{-1}, 3^{-1}$ $*$
					$(3\mathbb{A}_1, (6\mathbb{A}_1)_{II}) \subset 3\mathbb{A}_3$	19	4_5^{+3} $*$
					$(3\mathbb{A}_1, 8\mathbb{A}_1) \subset 11\mathbb{A}_1$	19	$2_1^{+1}, 4_2^{-2}, 3^{-1}$ $*$
					$(3\mathbb{A}_1, 12\mathbb{A}_1) \subset 15\mathbb{A}_1$	19	$2_7^{+1}, 4_7^{+1}, 8_7^{+1}$ $*$
					$(4\mathbb{A}_1, 4\mathbb{A}_1) \subset 8\mathbb{A}_1$ o	19	$2_{II}^{+2}, 4_1^{+1}, 3^{+2}$ $*$
					$(4\mathbb{A}_1, (6\mathbb{A}_1)_I) \subset 10\mathbb{A}_1$	19	$2_6^{+2}, 8_1^{+1}, 3^{+1}$ $*$
					$(4\mathbb{A}_1, (6\mathbb{A}_1)_{II}) \subset 10\mathbb{A}_1$	19	$2_{II}^{+2}, 8_3^{-1}, 3^{+1}$
					$(4\mathbb{A}_1, 8\mathbb{A}_1) \subset 12\mathbb{A}_1$ o	19	$8_1^{+1}, 3^{+2}$ $*$
					$(4\mathbb{A}_1, 8\mathbb{A}_1) \subset 4\mathbb{A}_3$	19	$4_1^{+1}, 3^{+2}$ $*$
					$(4\mathbb{A}_1, 12\mathbb{A}_1) \subset 16\mathbb{A}_1$ o	19	$2_{II}^{+2}, 4_3^{-1}, 3^{-1}$ $*$
					$(4\mathbb{A}_1, 12\mathbb{A}_1) \subset 4\mathbb{D}_4$	19	$4_3^{-1}, 3^{-1}$ $*$
					$(4\mathbb{A}_1, 6\mathbb{A}_2) \subset 4\mathbb{A}_1 \amalg 6\mathbb{A}_2$	19	$8_3^{-1}, 3^{+1}$ $*$
					$((6\mathbb{A}_1)_I, (6\mathbb{A}_1)_{II}) \subset 12\mathbb{A}_1$ o	19	4_5^{+3} $*$
					$((6\mathbb{A}_1)_I, 8\mathbb{A}_1) \subset 14\mathbb{A}_1$	19	$2_4^{-2}, 4_7^{+1}, 3^{+1}$ $*$
					$((6\mathbb{A}_1)_I, 12\mathbb{A}_1) \subset 6\mathbb{A}_3$	19	$2_6^{-2}, 4_3^{-1}$ $*$
39	32	27	2^4C_2	17	$(4\mathbb{A}_1, 4\mathbb{A}_1) \subset 8\mathbb{A}_1$ o	19	$4_4^{-2}, 8_5^{-1}$ $*$
					$(4\mathbb{A}_1, 8\mathbb{A}_1) \subset 12\mathbb{A}_1$ o	19	4_5^{+3} $*$
					$(8\mathbb{A}_1, 8\mathbb{A}_1) \subset 16\mathbb{A}_1$ o	19	$2_{II}^{-2}, 8_5^{-1}$ $*$
40	32	49	$Q_8 * Q_8$	17	$(8\mathbb{A}_1, 8\mathbb{A}_1) \subset 16\mathbb{A}_1$ o	19	4_5^{+3} $*$
49	48	50	2^4C_3	17	$(4\mathbb{A}_1, 4\mathbb{A}_1) \subset 8\mathbb{A}_1$ o	19	$2_{II}^{-2}, 8_7^{+1}, 3^{-1}$ $*$
					$(4\mathbb{A}_1, 12\mathbb{A}_1) \subset 16\mathbb{A}_1$ o	19	$2_{II}^{-2}, 8_5^{-1}$ $*$

Table 3: Types and lattices S of degenerations of codimension ≥ 2 of Kählerian K3 surfaces with symplectic automorphism group D_8.

n	$\lvert G\rvert$	i	G	rk S_G	Deg	rk S	q_S
10	8	3	D_8	15	$(\mathbb{A}_1,\mathbb{A}_1)\subset 2\mathbb{A}_1\ o$	17	$4_7^{+5}\ *$
					$(\mathbb{A}_1,(2\mathbb{A}_1)_I)\subset 3\mathbb{A}_1$	17	$2_7^{+1},4_0^{+4}$
					$(\mathbb{A}_1,(2\mathbb{A}_1)_{II})\subset \mathbb{A}_3$	17	$4_7^{+5}\ *$
					$(\mathbb{A}_1,4\mathbb{A}_1)\subset 5\mathbb{A}_1$	17	$2_7^{+1},4_1^{+3},8_7^{+1}$
					$(\mathbb{A}_1,8\mathbb{A}_1)\subset 9\mathbb{A}_1$	17	$2_1^{+1},4_6^{+4}$
					$(\mathbb{A}_1,2\mathbb{A}_2)\subset \mathbb{A}_1\amalg 2\mathbb{A}_2$	17	$2_7^{+1},4_{II}^{+4}\ *$
					$((2\mathbb{A}_1)_I,(2\mathbb{A}_1)_{II})\subset 4\mathbb{A}_1$	17	$4_7^{+5}\ *$
					$((2\mathbb{A}_1)_I,4\mathbb{A}_1)\subset (6\mathbb{A}_1)_I$	17	$2_6^{+2},4_0^{+2},8_1^{+1}$
					$((2\mathbb{A}_1)_I,4\mathbb{A}_1)\subset (6\mathbb{A}_1)_{II}$	17	$2_6^{+2},4_{II}^{+2},8_1^{+1}$
					$((2\mathbb{A}_1)_{II},4\mathbb{A}_1)\subset 6\mathbb{A}_1$	17	$2_{II}^{+2},4_{II}^{+2},8_7^{+1}$
					$((2\mathbb{A}_1)_I,4\mathbb{A}_1)\subset 2\mathbb{A}_3$	17	$2_0^{+2},4_7^{+3}\ *$
					$((2\mathbb{A}_1)_I,8\mathbb{A}_1)\subset 10\mathbb{A}_1$	17	$2_2^{+2},4_5^{+3}\ *$
					$((4\mathbb{A}_1,4\mathbb{A}_1)\subset 8\mathbb{A}_1)_I$	17	$4_7^{+1},8_0^{+2}\ *$
					$((4\mathbb{A}_1,4\mathbb{A}_1)\subset 8\mathbb{A}_1)_{II}\ o$	17	$2_{II}^{+2},4_7^{+3}\ *$
					$(4\mathbb{A}_1,4\mathbb{A}_1)\subset 4\mathbb{A}_2$	17	$4_1^{-3},3^{+1}\ *$
					$(4\mathbb{A}_1,8\mathbb{A}_1)\subset 12\mathbb{A}_1$	17	$4_6^{+2},8_1^{+1}\ *$
					$(4\mathbb{A}_1,8\mathbb{A}_1)\subset 4\mathbb{A}_3$	17	$4_7^{+3}\ *$
					$(4\mathbb{A}_1,2\mathbb{A}_2)\subset 4\mathbb{A}_1\amalg 2\mathbb{A}_2$	17	$4_{II}^{-2},8_3^{-1}\ *$
					$(\mathbb{A}_1,\mathbb{A}_1,(2\mathbb{A}_1)_I)\subset 4\mathbb{A}_1\ o$	18	$4_6^{+4}\ *$
					$(\mathbb{A}_1,\mathbb{A}_1,4\mathbb{A}_1)\subset 6\mathbb{A}_1\ o$	18	$4_7^{+3},8_7^{+1}\ *$
					$(\mathbb{A}_1,\mathbb{A}_1,8\mathbb{A}_1)\subset 10\mathbb{A}_1\ o$	18	4_6^{+4}
					$(\mathbb{A}_1,(2\mathbb{A}_1)_{II},(2\mathbb{A}_1)_I)\subset \mathbb{A}_3\amalg 2\mathbb{A}_1$	18	$4_6^{+4}\ *$
					$\begin{pmatrix}\mathbb{A}_1 & 3\mathbb{A}_1 & 5\mathbb{A}_1\\ & (2\mathbb{A}_1)_I & (6\mathbb{A}_1)_I\\ & & 4\mathbb{A}_1\end{pmatrix}\subset 7\mathbb{A}_1$	18	$2_7^{+1},4_6^{+2},8_5^{-1}$
					$(\mathbb{A}_1,(2\mathbb{A}_1)_I,4\mathbb{A}_1)\subset \mathbb{A}_1\amalg 2\mathbb{A}_3$	18	$2_7^{+1},4_7^{+3}\ *$
					$(\mathbb{A}_1,(2\mathbb{A}_1)_{II},4\mathbb{A}_1)\subset \mathbb{A}_3\amalg 4\mathbb{A}_1$	18	$4_5^{+3},8_1^{+1}$
					$(\mathbb{A}_1,(2\mathbb{A}_1)_I,8\mathbb{A}_1)\subset 11\mathbb{A}_1$	18	$2_7^{+1},4_7^{+3}\ *$
					$\begin{pmatrix}\mathbb{A}_1 & 5\mathbb{A}_1 & 5\mathbb{A}_1\\ & 4\mathbb{A}_1 & (8\mathbb{A}_1)_I\\ & & 4\mathbb{A}_1\end{pmatrix}\subset 9\mathbb{A}_1$	18	$2_7^{+1},4_7^{+1},8_4^{-2}$
					$(\mathbb{A}_1,4\mathbb{A}_1,4\mathbb{A}_1)\subset \mathbb{A}_1\amalg 4\mathbb{A}_2$	18	$2_7^{+1},4_1^{+3},3^{+1}\ *$

n	G	$\mathrm{rk}\, S_G$	Deg	$\mathrm{rk}\, S$	q_S
10	D_8	15	$(\mathbb{A}_1, 4\mathbb{A}_1, 8\mathbb{A}_1) \subset 13\mathbb{A}_1$	18	$2_7^{+1}, 4_0^{+2}, 8_7^{+1}$
			$(\mathbb{A}_1, 4\mathbb{A}_1, 8\mathbb{A}_1) \subset \mathbb{A}_1 \amalg 4\mathbb{A}_3$	18	$2_7^{+1}, 4_7^{+3}$
			$(\mathbb{A}_1, 4\mathbb{A}_1, 2\mathbb{A}_2) \subset 5\mathbb{A}_1 \amalg 2\mathbb{A}_2$	18	$2_3^{-1}, 4_{II}^{+2}, 8_3^{-1}$
			$\begin{pmatrix} (2\mathbb{A}_1)_I & 4\mathbb{A}_1 & (6\mathbb{A}_1)_I \\ & (2\mathbb{A}_1)_{II} & 6\mathbb{A}_1 \\ & & 4\mathbb{A}_1 \end{pmatrix} \subset 8\mathbb{A}_1$	18	$4_5^{+3}, 8_1^{+1}$
			$\begin{pmatrix} (2\mathbb{A}_1)_I & 4\mathbb{A}_1 & (6\mathbb{A}_1)_{II} \\ & (2\mathbb{A}_1)_{II} & 6\mathbb{A}_1 \\ & & 4\mathbb{A}_1 \end{pmatrix} \subset 8\mathbb{A}_1$	18	$4_7^{+3}, 8_7^{+1}$ *
			$\begin{pmatrix} (2\mathbb{A}_1)_{II} & 4\mathbb{A}_1 & 6\mathbb{A}_1 \\ & (2\mathbb{A}_1)_I & 2\mathbb{A}_3 \\ & & 4\mathbb{A}_1 \end{pmatrix} \subset 2\mathbb{A}_1 \amalg 2\mathbb{A}_3$	18	4_6^{+4} *
			$\begin{pmatrix} (2\mathbb{A}_1)_I & (6\mathbb{A}_1)_I & (6\mathbb{A}_1)_{II} \\ & 4\mathbb{A}_1 & (8\mathbb{A}_1)_I \\ & & 4\mathbb{A}_1 \end{pmatrix} \subset 10\mathbb{A}_1$	18	$2_6^{+2}, 8_0^{+2}$
			$\begin{pmatrix} (2\mathbb{A}_1)_I & (6\mathbb{A}_1)_I & (6\mathbb{A}_1)_I \\ & 4\mathbb{A}_1 & (8\mathbb{A}_1)_{II} \\ & & 4\mathbb{A}_1 \end{pmatrix} \subset 10\mathbb{A}_1 \ o$	18	4_6^{+4}
			$\begin{pmatrix} (2\mathbb{A}_1)_{II} & 6\mathbb{A}_1 & 6\mathbb{A}_1 \\ & 4\mathbb{A}_1 & (8\mathbb{A}_1)_I \\ & & 4\mathbb{A}_1 \end{pmatrix} \subset 10\mathbb{A}_1$	18	$2_{II}^{+2}, 8_6^{+2}$
			$\begin{pmatrix} (2\mathbb{A}_1)_{II} & 6\mathbb{A}_1 & 6\mathbb{A}_1 \\ & 4\mathbb{A}_1 & (8\mathbb{A}_1)_{II} \\ & & 4\mathbb{A}_1 \end{pmatrix} \subset 10\mathbb{A}_1 \ o$	18	4_6^{+4}
			$\begin{pmatrix} (2\mathbb{A}_1)_I & (6\mathbb{A}_1)_I & (6\mathbb{A}_1)_I \\ & 4\mathbb{A}_1 & 4\mathbb{A}_2 \\ & & 4\mathbb{A}_1 \end{pmatrix} \subset 2\mathbb{A}_1 \amalg 4\mathbb{A}_2$	18	$2_2^{+2}, 4_6^{+2}, 3^{+1}$ *
			$\begin{pmatrix} (2\mathbb{A}_1)_{II} & 6\mathbb{A}_1 & 6\mathbb{A}_1 \\ & 4\mathbb{A}_1 & 4\mathbb{A}_2 \\ & & 4\mathbb{A}_1 \end{pmatrix} \subset 2\mathbb{A}_1 \amalg 4\mathbb{A}_2$	18	$2_{II}^{+2}, 4_{II}^{-2}, 3^{+1}$
			$\begin{pmatrix} (2\mathbb{A}_1)_I & 2\mathbb{A}_3 & (6\mathbb{A}_1)_I \\ & 4\mathbb{A}_1 & (8\mathbb{A}_1)_I \\ & & 4\mathbb{A}_1 \end{pmatrix} \subset 2\mathbb{A}_3 \amalg 4\mathbb{A}_1$	18	$2_0^{+2}, 4_7^{+1}, 8_3^{-1}$
			$\begin{pmatrix} (2\mathbb{A}_1)_I & 2\mathbb{A}_3 & (6\mathbb{A}_1)_{II} \\ & 4\mathbb{A}_1 & 4\mathbb{A}_2 \\ & & 4\mathbb{A}_1 \end{pmatrix} \subset 2\mathbb{A}_5$	18	$2_2^{-2}, 4_{II}^{+2}$ *

n	G	rk S_G	Deg	rk S	q_S
10	D_8	15	$\begin{pmatrix} (2\mathbb{A}_1)_I & (6\mathbb{A}_1)_I & 10\mathbb{A}_1 \\ & 4\mathbb{A}_1 & 12\mathbb{A}_1 \\ & & 8\mathbb{A}_1 \end{pmatrix} \subset 14\mathbb{A}_1$	18	$2_2^{-2}, 4_1^{+1}, 8_3^{-1}$ $*$
			$\begin{pmatrix} (2\mathbb{A}_1)_I & (6\mathbb{A}_1)_I & 10\mathbb{A}_1 \\ & 4\mathbb{A}_1 & 4\mathbb{A}_3 \\ & & 8\mathbb{A}_1 \end{pmatrix} \subset 2\mathbb{A}_1 \amalg 4\mathbb{A}_3$	18	$2_2^{-2}, 4_0^{+2}$
			$\begin{pmatrix} (2\mathbb{A}_1)_I & (6\mathbb{A}_1)_{II} & 10\mathbb{A}_1 \\ & 4\mathbb{A}_1 & 4\mathbb{A}_3 \\ & & 8\mathbb{A}_1 \end{pmatrix} \subset 2\mathbb{A}_1 \amalg 4\mathbb{A}_3$	18	$2_2^{-2}, 4_{II}^{-2}$ $*$
			$((2\mathbb{A}_1)_I, 4\mathbb{A}_1, 8\mathbb{A}_1) \subset 2\mathbb{A}_3 \amalg 8\mathbb{A}_1$	18	$2_4^{-2}, 4_6^{-2}$ $*$
			$\begin{pmatrix} 4\mathbb{A}_1 & (8\mathbb{A}_1)_I & (8\mathbb{A}_1)_{II} \\ & 4\mathbb{A}_1 & (8\mathbb{A}_1)_I \\ & & 4\mathbb{A}_1 \end{pmatrix} \subset 12\mathbb{A}_1$ o	18	$2_{II}^{-2}, 4_1^{+1}, 8_5^{-1}$ $*$
			$\begin{pmatrix} 4\mathbb{A}_1 & (8\mathbb{A}_1)_I & (8\mathbb{A}_1)_I \\ & 4\mathbb{A}_1 & 4\mathbb{A}_2 \\ & & 4\mathbb{A}_1 \end{pmatrix} \subset 4\mathbb{A}_1 \amalg 4\mathbb{A}_2$	18	$4_7^{+1}, 8_5^{-1}, 3^{+1}$ $*$
			$\begin{pmatrix} 4\mathbb{A}_1 & 4\mathbb{A}_2 & (8\mathbb{A}_1)_{II} \\ & 4\mathbb{A}_1 & 4\mathbb{A}_2 \\ & & 4\mathbb{A}_1 \end{pmatrix} \subset 4\mathbb{A}_3$ o	18	$2_{II}^{-2}, 4_2^{-2}$ $*$
			$\begin{pmatrix} 4\mathbb{A}_1 & (8\mathbb{A}_1)_{II} & 12\mathbb{A}_1 \\ & 4\mathbb{A}_1 & 12\mathbb{A}_1 \\ & & 8\mathbb{A}_1 \end{pmatrix} \subset 16\mathbb{A}_1$ o	18	$2_{II}^{+2}, 4_6^{+2}$ $*$
			$\begin{pmatrix} 4\mathbb{A}_1 & (8\mathbb{A}_1)_I & 12\mathbb{A}_1 \\ & 4\mathbb{A}_1 & 4\mathbb{A}_3 \\ & & 8\mathbb{A}_1 \end{pmatrix} \subset 4\mathbb{A}_1 \amalg 4\mathbb{A}_3$	18	$4_5^{-1}, 8_5^{-1}$ $*$
			$\begin{pmatrix} 4\mathbb{A}_1 & 4\mathbb{A}_2 & 12\mathbb{A}_1 \\ & 4\mathbb{A}_1 & 4\mathbb{A}_3 \\ & & 8\mathbb{A}_1 \end{pmatrix} \subset 4\mathbb{D}_4$	18	4_6^{+2} $*$
			$(4\mathbb{A}_1, 4\mathbb{A}_1, 2\mathbb{A}_2) \subset (8\mathbb{A}_1)_I \amalg 2\mathbb{A}_2$	18	8_6^{+2} $*$
			$(4\mathbb{A}_1, 4\mathbb{A}_1, 2\mathbb{A}_2) \subset 4\mathbb{A}_2 \amalg 2\mathbb{A}_2$	18	$4_{II}^{-2}, 3^{+1}$ $*$

n	G	Deg	rk S	q_S
10	D_8	$\begin{pmatrix} \mathbb{A}_1 & 2\mathbb{A}_1 & 3\mathbb{A}_1 & 5\mathbb{A}_1 \\ & \mathbb{A}_1 & 3\mathbb{A}_1 & 5\mathbb{A}_1 \\ & & (2\mathbb{A}_1)_I & (6\mathbb{A}_1)_I \\ & & & 4\mathbb{A}_1 \end{pmatrix} \subset 8\mathbb{A}_1 \ o$	19	$4_4^{-2}, 8_5^{-1} \ *$
		$\begin{pmatrix} \mathbb{A}_1 & 2\mathbb{A}_1 & 5\mathbb{A}_1 & 5\mathbb{A}_1 \\ & \mathbb{A}_1 & 5\mathbb{A}_1 & 5\mathbb{A}_1 \\ & & 4\mathbb{A}_1 & (8\mathbb{A}_1)_I \\ & & & 4\mathbb{A}_1 \end{pmatrix} \subset 10\mathbb{A}_1 \ o$	19	$4_5^{-1}, 8_4^{-2} \ *$
		$(\mathbb{A}_1,\, \mathbb{A}_1,\, (2\mathbb{A}_1)_I,\, 8\mathbb{A}_1) \subset 12\mathbb{A}_1 \ o$	19	$4_5^{+3} \ *$
		$(\mathbb{A}_1,\, \mathbb{A}_1,\, 4\mathbb{A}_1,\, 8\mathbb{A}_1) \subset 14\mathbb{A}_1 \ o$	19	$4_4^{-2}, 8_5^{-1}$
		$(\mathbb{A}_1,\, \mathbb{A}_1,\, 4\mathbb{A}_1,\, 8\mathbb{A}_1) \subset 2\mathbb{A}_1 \amalg 4\mathbb{A}_3 \ \ o$	19	$4_5^{+3} \ *$
		$\begin{pmatrix} \mathbb{A}_1 & \mathbb{A}_3 & 3\mathbb{A}_1 & 5\mathbb{A}_1 \\ & (2\mathbb{A}_1)_{II} & 4\mathbb{A}_1 & 8\mathbb{A}_1 \\ & & (2\mathbb{A}_1)_I & (6\mathbb{A}_1)_I \\ & & & 4\mathbb{A}_1 \end{pmatrix} \subset \mathbb{A}_3 \amalg 6\mathbb{A}_1$	19	$4_4^{-2}, 8_5^{-1}$
		$\begin{pmatrix} \mathbb{A}_1 & \mathbb{A}_3 & 3\mathbb{A}_1 & 5\mathbb{A}_1 \\ & (2\mathbb{A}_1)_{II} & 4\mathbb{A}_1 & 6\mathbb{A}_1 \\ & & (2\mathbb{A}_1)_I & 2\mathbb{A}_3 \\ & & & 4\mathbb{A}_1 \end{pmatrix} \subset 3\mathbb{A}_3 \ o$	19	$4_5^{+3} \ *$
		$\begin{pmatrix} \mathbb{A}_1 & 3\mathbb{A}_1 & 5\mathbb{A}_1 & 5\mathbb{A}_1 \\ & (2\mathbb{A}_1)_I & 2\mathbb{A}_3 & (6\mathbb{A}_1)_I \\ & & 4\mathbb{A}_1 & (8\mathbb{A}_1)_I \\ & & & 4\mathbb{A}_1 \end{pmatrix} \subset \mathbb{A}_1 \amalg 2\mathbb{A}_3 \amalg 4\mathbb{A}_1$	19	$2_3^{-1}, 4_5^{-1}, 8_1^{+1} \ *$
		$\begin{pmatrix} \mathbb{A}_1 & \mathbb{A}_3 & 5\mathbb{A}_1 & 5\mathbb{A}_1 \\ & (2\mathbb{A}_1)_{II} & 6\mathbb{A}_1 & 6\mathbb{A}_1 \\ & & 4\mathbb{A}_1 & (8\mathbb{A}_1)_I \\ & & & 4\mathbb{A}_1 \end{pmatrix} \subset \mathbb{A}_3 \amalg 8\mathbb{A}_1$	19	$4_7^{+1}, 8_6^{+2}$
		$\begin{pmatrix} \mathbb{A}_1 & 3\mathbb{A}_1 & 5\mathbb{A}_1 & 5\mathbb{A}_1 \\ & (2\mathbb{A}_1)_I & (6\mathbb{A}_1)_I & (6\mathbb{A}_1)_I \\ & & 4\mathbb{A}_1 & 4\mathbb{A}_2 \\ & & & 4\mathbb{A}_1 \end{pmatrix} \subset 3\mathbb{A}_1 \amalg 4\mathbb{A}_2$	19	$2_3^{-1}, 4_0^{+2}, 3^{+1} \ *$
		$(\mathbb{A}_1,\, (2\mathbb{A}_1)_{II},\, 4\mathbb{A}_1,\, 4\mathbb{A}_1) \subset \mathbb{A}_3 \amalg 4\mathbb{A}_2$	19	$4_7^{-3}, 3^{+1} \ *$
		$\begin{pmatrix} \mathbb{A}_1 & 3\mathbb{A}_1 & 5\mathbb{A}_1 & 9\mathbb{A}_1 \\ & (2\mathbb{A}_1)_I & (6\mathbb{A}_1)_I & 10\mathbb{A}_1 \\ & & 4\mathbb{A}_1 & 12\mathbb{A}_1 \\ & & & 8\mathbb{A}_1 \end{pmatrix} \subset 15\mathbb{A}_1 \ \ o$	19	$2_1^{+1}, 4_5^{-1}, 8_3^{-1} \ *$

n	G	*Deg*	rk S	q_S
10	D_8	$(\mathbb{A}_1,\ (2\mathbb{A}_1)_I,\ 4\mathbb{A}_1,\ 8\mathbb{A}_1) \subset \mathbb{A}_1 \amalg 2\mathbb{A}_3, \amalg 8\mathbb{A}_1$	19	$2_3^{-1}, 4_6^{-2}$ $*$
		$\begin{pmatrix} \mathbb{A}_1 & 3\mathbb{A}_1 & 5\mathbb{A}_1 & 9\mathbb{A}_1 \\ & (2\mathbb{A}_1)_I & (6\mathbb{A}_1)_I & 10\mathbb{A}_1 \\ & & 4\mathbb{A}_1 & 4\mathbb{A}_3 \\ & & & 8\mathbb{A}_1 \end{pmatrix} \subset 3\mathbb{A}_1 \amalg 4\mathbb{A}_3$	19	$2_5^{-1}, 4_4^{-2}$
		$\begin{pmatrix} \mathbb{A}_1 & 5\mathbb{A}_1 & 5\mathbb{A}_1 & 5\mathbb{A}_1 \\ & 4\mathbb{A}_1 & (8\mathbb{A}_1)_I & (8\mathbb{A}_1)_I \\ & & 4\mathbb{A}_1 & 4\mathbb{A}_2 \\ & & & 4\mathbb{A}_1 \end{pmatrix} \subset 5\mathbb{A}_1 \amalg 4\mathbb{A}_2$	19	$2_1^{+1}, 4_7^{+1}, 8_3^{-1}, 3^{+1}$
		$\begin{pmatrix} \mathbb{A}_1 & 5\mathbb{A}_1 & 5\mathbb{A}_1 & 9\mathbb{A}_1 \\ & 4\mathbb{A}_1 & (8\mathbb{A}_1)_I & 12\mathbb{A}_1 \\ & & 4\mathbb{A}_1 & 4\mathbb{A}_3 \\ & & & 8\mathbb{A}_1 \end{pmatrix} \subset 5\mathbb{A}_1 \amalg 4\mathbb{A}_3$	19	$2_7^{+1}, 4_5^{-1}, 8_5^{-1}$ $*$
		$\begin{pmatrix} \mathbb{A}_1 & 5\mathbb{A}_1 & 5\mathbb{A}_1 & 9\mathbb{A}_1 \\ & 4\mathbb{A}_1 & 4\mathbb{A}_2 & 12\mathbb{A}_1 \\ & & 4\mathbb{A}_1 & 4\mathbb{A}_3 \\ & & & 8\mathbb{A}_1 \end{pmatrix} \subset \mathbb{A}_1 \amalg 4\mathbb{D}_4$	19	$2_7^{+1}, 4_6^{+2}$
		$\begin{pmatrix} \mathbb{A}_1 & 5\mathbb{A}_1 & 5\mathbb{A}_1 & \mathbb{A}_1 \amalg 2\mathbb{A}_2 \\ & 4\mathbb{A}_1 & (8\mathbb{A}_1)_I & 4\mathbb{A}_1 \amalg 2\mathbb{A}_2 \\ & & 4\mathbb{A}_1 & 4\mathbb{A}_1 \amalg 2\mathbb{A}_2 \\ & & & 2\mathbb{A}_2 \end{pmatrix} \subset 9\mathbb{A}_1 \amalg 2\mathbb{A}_2$	19	$2_7^{+1}, 8_6^{+2}$
		$(\mathbb{A}_1,\ 4\mathbb{A}_1,\ 4\mathbb{A}_1,\ 2\mathbb{A}_2) \subset \mathbb{A}_1 \amalg 6\mathbb{A}_2\ o$	19	$2_7^{+1}, 4_{II}^{+2}, 3^{+1}$ $*$
		$\begin{pmatrix} (2\mathbb{A}_1)_I & 4\mathbb{A}_1 & (6\mathbb{A}_1)_I & (6\mathbb{A}_1)_{II} \\ & (2\mathbb{A}_1)_{II} & 6\mathbb{A}_1 & 6\mathbb{A}_1 \\ & & 4\mathbb{A}_1 & (8\mathbb{A}_1)_I \\ & & & 4\mathbb{A}_1 \end{pmatrix} \subset 12\mathbb{A}_1$	19	$4_7^{+1}, 8_6^{+2}$
		$\begin{pmatrix} (2\mathbb{A}_1)_I & 4\mathbb{A}_1 & (6\mathbb{A}_1)_I & (6\mathbb{A}_1)_I \\ & (2\mathbb{A}_1)_{II} & 6\mathbb{A}_1 & 6\mathbb{A}_1 \\ & & 4\mathbb{A}_1 & (8\mathbb{A}_1)_{II} \\ & & & 4\mathbb{A}_1 \end{pmatrix} \subset 12\mathbb{A}_1\ o$	19	4_5^{+3} $*$
		$\begin{pmatrix} (2\mathbb{A}_1)_I & 4\mathbb{A}_1 & (6\mathbb{A}_1)_I & (6\mathbb{A}_1)_I \\ & (2\mathbb{A}_1)_{II} & 6\mathbb{A}_1 & 6\mathbb{A}_1 \\ & & 4\mathbb{A}_1 & 4\mathbb{A}_2 \\ & & & 4\mathbb{A}_1 \end{pmatrix} \subset 4\mathbb{A}_1 \amalg 4\mathbb{A}_2$	19	$4_7^{-3}, 3^{+1}$ $*$

n	G	Deg	rk S	q_S
10	D_8	$\begin{pmatrix} (2\mathbb{A}_1)_{II} & 6\mathbb{A}_1 & 4\mathbb{A}_1 & 6\mathbb{A}_1 \\ & 4\mathbb{A}_1 & (6\mathbb{A}_1)_I & (8\mathbb{A}_1)_I \\ & & (2\mathbb{A}_1)_I & 2\mathbb{A}_3 \\ & & & 4\mathbb{A}_1 \end{pmatrix} \subset 6\mathbb{A}_1 \amalg 2\mathbb{A}_3$	19	$4_6^{+2}, 8_7^{+1}$
		$\begin{pmatrix} (2\mathbb{A}_1)_{II} & 4\mathbb{A}_1 & 6\mathbb{A}_1 & 6\mathbb{A}_1 \\ & (2\mathbb{A}_1)_I & 2\mathbb{A}_3 & (6\mathbb{A}_1)_{II} \\ & & 4\mathbb{A}_1 & 4\mathbb{A}_2 \\ & & & 4\mathbb{A}_1 \end{pmatrix} \subset 2\mathbb{A}_1 \amalg 2\mathbb{A}_5$	19	4_5^{+3} *
		$\begin{pmatrix} (2\mathbb{A}_1)_I & (6\mathbb{A}_1)_{II} & (6\mathbb{A}_1)_I & (6\mathbb{A}_1)_I \\ & 4\mathbb{A}_1 & (8\mathbb{A}_1)_I & (8\mathbb{A}_1)_I \\ & & 4\mathbb{A}_1 & (8\mathbb{A}_1)_{II} \\ & & & 4\mathbb{A}_1 \end{pmatrix} \subset 14\mathbb{A}_1 \ o$	19	$4_6^{+2}, 8_7^{+1}$
		$\begin{pmatrix} (2\mathbb{A}_1)_{II} & 6\mathbb{A}_1 & 6\mathbb{A}_1 & 6\mathbb{A}_1 \\ & 4\mathbb{A}_1 & (8\mathbb{A}_1)_I & (8\mathbb{A}_1)_I \\ & & 4\mathbb{A}_1 & (8\mathbb{A}_1)_{II} \\ & & & 4\mathbb{A}_1 \end{pmatrix} \subset 14\mathbb{A}_1 \ o$	19	$4_4^{-2}, 8_5^{-1}$
		$\begin{pmatrix} (2\mathbb{A}_1)_I & (6\mathbb{A}_1)_{II} & (6\mathbb{A}_1)_I & (6\mathbb{A}_1)_I \\ & 4\mathbb{A}_1 & (8\mathbb{A}_1)_I & (8\mathbb{A}_1)_I \\ & & 4\mathbb{A}_1 & 4\mathbb{A}_2 \\ & & & 4\mathbb{A}_1 \end{pmatrix} \subset 6\mathbb{A}_1 \amalg 4\mathbb{A}_2$	19	$2_2^{-2}, 8_1^{+1}, 3^{+1}$ *
		$\begin{pmatrix} (2\mathbb{A}_1)_{II} & 6\mathbb{A}_1 & 6\mathbb{A}_1 & 6\mathbb{A}_1 \\ & 4\mathbb{A}_1 & (8\mathbb{A}_1)_I & (8\mathbb{A}_1)_I \\ & & 4\mathbb{A}_1 & 4\mathbb{A}_2 \\ & & & 4\mathbb{A}_1 \end{pmatrix} \subset 6\mathbb{A}_1 \amalg 4\mathbb{A}_2$	19	$2_{II}^{+2}, 8_3^{-1}, 3^{+1}$
		$\begin{pmatrix} (2\mathbb{A}_1)_I & (6\mathbb{A}_1)_I & (6\mathbb{A}_1)_I & (6\mathbb{A}_1)_I \\ & 4\mathbb{A}_1 & 4\mathbb{A}_2 & (8\mathbb{A}_1)_{II} \\ & & 4\mathbb{A}_1 & 4\mathbb{A}_2 \\ & & & 4\mathbb{A}_1 \end{pmatrix} \subset 2\mathbb{A}_1 \amalg 4\mathbb{A}_3 \ o$	19	4_5^{+3} *
		$\begin{pmatrix} (2\mathbb{A}_1)_{II} & 6\mathbb{A}_1 & 6\mathbb{A}_1 & 6\mathbb{A}_1 \\ & 4\mathbb{A}_1 & 4\mathbb{A}_2 & (8\mathbb{A}_1)_{II} \\ & & 4\mathbb{A}_1 & 4\mathbb{A}_2 \\ & & & 4\mathbb{A}_1 \end{pmatrix} \subset 2\mathbb{A}_1 \amalg 4\mathbb{A}_3 \ o$	19	4_5^{+3} *
		$\begin{pmatrix} (2\mathbb{A}_1)_I & 2\mathbb{A}_3 & (6\mathbb{A}_1)_I & (6\mathbb{A}_1)_I \\ & 4\mathbb{A}_1 & (8\mathbb{A}_1)_I & (8\mathbb{A}_1)_I \\ & & 4\mathbb{A}_1 & 4\mathbb{A}_2 \\ & & & 4\mathbb{A}_1 \end{pmatrix} \subset 2\mathbb{A}_3 \amalg 4\mathbb{A}_2$	19	$2_0^{+2}, 4_7^{+1}, 3^{+1}$ *

n	G	Deg	rk S	q_S
10	D_8	$\begin{pmatrix} 4\mathbb{A}_1 & (6\mathbb{A}_1)_I & (8\mathbb{A}_1)_I & (8\mathbb{A}_1)_I \\ & (2\mathbb{A}_1)_I & 2\mathbb{A}_3 & (6\mathbb{A}_1)_{II} \\ & & 4\mathbb{A}_1 & 4\mathbb{A}_2 \\ & & & 4\mathbb{A}_1 \end{pmatrix} \subset 4\mathbb{A}_1 \amalg 2\mathbb{A}_5$	19	$2_6^{+2}, 8_7^{+1}$
		$\begin{pmatrix} (2\mathbb{A}_1)_I & (6\mathbb{A}_1)_I & (6\mathbb{A}_1)_{II} & 10\mathbb{A}_1 \\ & 4\mathbb{A}_1 & (8\mathbb{A}_1)_I & 12\mathbb{A}_1 \\ & & 4\mathbb{A}_1 & 4\mathbb{A}_3 \\ & & & 8\mathbb{A}_1 \end{pmatrix} \subset 6\mathbb{A}_1 \amalg 4\mathbb{A}_3$	19	$2_2^{-2}, 8_3^{-1}$ *
		$\begin{pmatrix} (2\mathbb{A}_1)_I & 2\mathbb{A}_3 & (6\mathbb{A}_1)_I & 10\mathbb{A}_1 \\ & 4\mathbb{A}_1 & (8\mathbb{A}_1)_I & 12\mathbb{A}_1 \\ & & 4\mathbb{A}_1 & 4\mathbb{A}_3 \\ & & & 8\mathbb{A}_1 \end{pmatrix} \subset 6\mathbb{A}_3 \quad o$	19	$2_4^{-2}, 4_5^{-1}$ *
		$\begin{pmatrix} (2\mathbb{A}_1)_I & (6\mathbb{A}_1)_I & (6\mathbb{A}_1)_I & 10\mathbb{A}_1 \\ & 4\mathbb{A}_1 & 4\mathbb{A}_2 & 12\mathbb{A}_1 \\ & & 4\mathbb{A}_1 & 4\mathbb{A}_3 \\ & & & 8\mathbb{A}_1 \end{pmatrix} \subset 2\mathbb{A}_1 \amalg 4\mathbb{D}_4$	19	$2_6^{+2}, 4_7^{+1}$ *
		$\begin{pmatrix} 4\mathbb{A}_1 & (8\mathbb{A}_1)_{II} & (8\mathbb{A}_1)_I & (8\mathbb{A}_1)_I \\ & 4\mathbb{A}_1 & (8\mathbb{A}_1)_I & (8\mathbb{A}_1)_I \\ & & 4\mathbb{A}_1 & (8\mathbb{A}_1)_{II} \\ & & & 4\mathbb{A}_1 \end{pmatrix} \subset 16\mathbb{A}_1 \ o$	19	4_5^{+3} *
		$\begin{pmatrix} 4\mathbb{A}_1 & (8\mathbb{A}_1)_I & (8\mathbb{A}_1)_I & (8\mathbb{A}_1)_I \\ & 4\mathbb{A}_1 & 4\mathbb{A}_2 & (8\mathbb{A}_1)_{II} \\ & & 4\mathbb{A}_1 & 4\mathbb{A}_2 \\ & & & 4\mathbb{A}_1 \end{pmatrix} \subset 4\mathbb{A}_1 \amalg 4\mathbb{A}_3 \ o$	19	$2_{II}^{-2}, 8_5^{-1}$ *
		$\begin{pmatrix} 4\mathbb{A}_1 & 4\mathbb{A}_2 & (8\mathbb{A}_1)_I & (8\mathbb{A}_1)_I \\ & 4\mathbb{A}_1 & (8\mathbb{A}_1)_I & (8\mathbb{A}_1)_I \\ & & 4\mathbb{A}_1 & 4\mathbb{A}_2 \\ & & & 4\mathbb{A}_1 \end{pmatrix} \subset 8\mathbb{A}_2$	19	$4_1^{+1}, 3^{+2}$ *
		$\begin{pmatrix} 4\mathbb{A}_1 & (8\mathbb{A}_1)_I & (8\mathbb{A}_1)_I & 4\mathbb{A}_1 \amalg 2\mathbb{A}_2 \\ & 4\mathbb{A}_1 & 4\mathbb{A}_2 & 4\mathbb{A}_1 \amalg 2\mathbb{A}_2 \\ & & 4\mathbb{A}_1 & 4\mathbb{A}_1 \amalg 2\mathbb{A}_2 \\ & & & 2\mathbb{A}_2 \end{pmatrix} \subset 4\mathbb{A}_1 \amalg 6\mathbb{A}_2 \quad o$	19	$8_3^{-1}, 3^{+1}$ *

Table 4: Types and lattices S of degenerations of codimension ≥ 2 of Kählerian K3 surfaces with symplectic automorphism group $(C_2)^3$.

n	$\lvert G\rvert$	G	Deg	rk S	q_S
9	8	$(C_2)^3$	$((2\mathbb{A}_1, 2\mathbb{A}_1) \subset 4\mathbb{A}_1)_I$	16	$2_{II}^{+2}, 4_0^{+4}$ $*$
			$((2\mathbb{A}_1, 2\mathbb{A}_1) \subset 4\mathbb{A}_1)_{II}$ o	16	$2_{II}^{+4}, 4_{II}^{+2}$ $*$
			$(2\mathbb{A}_1, 4\mathbb{A}_1) \subset 6\mathbb{A}_1$	16	$2_{II}^{+4}, 4_7^{+1}, 8_1^{+1}$ $*$
			$(2\mathbb{A}_1, 4\mathbb{A}_1) \subset 2\mathbb{A}_3$	16	$2_{II}^{+4}, 4_{II}^{+2}$ $*$
			$(2\mathbb{A}_1, 8\mathbb{A}_1) \subset 10\mathbb{A}_1$	16	$2_{II}^{-4}, 4_4^{-2}$ $*$
			$(4\mathbb{A}_1, 4\mathbb{A}_1) \subset 8\mathbb{A}_1$	16	$2_{II}^{+4}, 4_{II}^{+2}$
			$(4\mathbb{A}_1, 8\mathbb{A}_1) \subset 4\mathbb{A}_3$	16	2_{II}^{+6} $*$
			$(8\mathbb{A}_1, 8\mathbb{A}_1) \subset 16\mathbb{A}_1$ o	16	2_{II}^{+6} $*$
			$\begin{pmatrix} 2\mathbb{A}_1 & (4\mathbb{A}_1)_I & (4\mathbb{A}_1)_I \\ & 2\mathbb{A}_1 & (4\mathbb{A}_1)_I \\ & & 2\mathbb{A}_1 \end{pmatrix} \subset 6\mathbb{A}_1$	17	4_7^{+5} $*$
			$((2\mathbb{A}_1, 2\mathbb{A}_1)_I, 4\mathbb{A}_1) \subset 8\mathbb{A}_1$	17	$2_{II}^{+2}, 4_6^{+2}, 8_1^{+1}$ $*$
			$((2\mathbb{A}_1, 2\mathbb{A}_1)_{II}, 4\mathbb{A}_1) \subset 8\mathbb{A}_1$ o	17	$2_{II}^{+4}, 8_7^{+1}$ $*$
			$\begin{pmatrix} 2\mathbb{A}_1 & (4\mathbb{A}_1)_I & 6\mathbb{A}_1 \\ & 2\mathbb{A}_1 & 2\mathbb{A}_3 \\ & & 4\mathbb{A}_1 \end{pmatrix} \subset 2\mathbb{A}_1 \amalg 2\mathbb{A}_3$	17	$2_{II}^{+2}, 4_7^{+3}$ $*$
			$((2\mathbb{A}_1, 2\mathbb{A}_1)_I, 8\mathbb{A}_1) \subset 12\mathbb{A}_1$	17	$2_{II}^{+2}, 4_7^{+3}$ $*$
			$(2\mathbb{A}_1, 4\mathbb{A}_1, 4\mathbb{A}_1) \subset 10\mathbb{A}_1$	17	$2_{II}^{+2}, 4_7^{+3}$ $*$
			$(2\mathbb{A}_1, 4\mathbb{A}_1, 4\mathbb{A}_1) \subset 2\mathbb{A}_3 \amalg 4\mathbb{A}_1$	17	$2_{II}^{+4}, 8_7^{+1}$ $*$
			$(2\mathbb{A}_1, 4\mathbb{A}_1, 8\mathbb{A}_1) \subset 2\mathbb{A}_1 \amalg 4\mathbb{A}_3$	17	$2_{II}^{+4}, 4_7^{+1}$ $*$
			$(4\mathbb{A}_1, 4\mathbb{A}_1, 4\mathbb{A}_1) \subset 12\mathbb{A}_1$	17	$2_{II}^{+4}, 8_7^{+1}$
			$\begin{pmatrix} 2\mathbb{A}_1 & (4\mathbb{A}_1)_I & (4\mathbb{A}_1)_I & (4\mathbb{A}_1)_I \\ & 2\mathbb{A}_1 & (4\mathbb{A}_1)_I & (4\mathbb{A}_1)_I \\ & & 2\mathbb{A}_1 & (4\mathbb{A}_1)_I \\ & & & 2\mathbb{A}_1 \end{pmatrix} \subset 8\mathbb{A}_1$ o	18	4_6^{+4} $*$
			$\begin{pmatrix} 2\mathbb{A}_1 & (4\mathbb{A}_1)_I & (4\mathbb{A}_1)_I & 6\mathbb{A}_1 \\ & 2\mathbb{A}_1 & (4\mathbb{A}_1)_I & 6\mathbb{A}_1 \\ & & 2\mathbb{A}_1 & 6\mathbb{A}_1 \\ & & & 4\mathbb{A}_1 \end{pmatrix} \subset 10\mathbb{A}_1$	18	$4_5^{+3}, 8_1^{+1}$ $*$
			$\begin{pmatrix} 2\mathbb{A}_1 & (4\mathbb{A}_1)_I & (4\mathbb{A}_1)_I & 6\mathbb{A}_1 \\ & 2\mathbb{A}_1 & (4\mathbb{A}_1)_I & 6\mathbb{A}_1 \\ & & 2\mathbb{A}_1 & 2\mathbb{A}_3 \\ & & & 4\mathbb{A}_1 \end{pmatrix} \subset 4\mathbb{A}_1 \amalg 2\mathbb{A}_3$	18	4_6^{+4} $*$

n	G	Deg	$\mathrm{rk}\, S$	q_S
9	$(C_2)^3$	$\begin{pmatrix} 2\mathbb{A}_1 & (4\mathbb{A}_1)_I & (4\mathbb{A}_1)_I & 10\mathbb{A}_1 \\ & 2\mathbb{A}_1 & (4\mathbb{A}_1)_I & 10\mathbb{A}_1 \\ & & 2\mathbb{A}_1 & 10\mathbb{A}_1 \\ & & & 8\mathbb{A}_1 \end{pmatrix} \subset 14\mathbb{A}_1$	18	4_6^{+4}
		$((2\mathbb{A}_1, 2\mathbb{A}_1)_I, 4\mathbb{A}_1, 4\mathbb{A}_1) \subset 12\mathbb{A}_1$	18	$4_6^{+4}\ *$
		$((2\mathbb{A}_1, 2\mathbb{A}_1)_{II}, 4\mathbb{A}_1, 4\mathbb{A}_1) \subset 12\mathbb{A}_1\ o$	18	$2_{II}^{-2}, 4_2^{-2}\ *$
		$\begin{pmatrix} 2\mathbb{A}_1 & 2\mathbb{A}_3 & (4\mathbb{A}_1)_I & 6\mathbb{A}_1 \\ & 4\mathbb{A}_1 & 6\mathbb{A}_1 & 8\mathbb{A}_1 \\ & & 2\mathbb{A}_1 & 6\mathbb{A}_1 \\ & & & 4\mathbb{A}_1 \end{pmatrix} \subset 2\mathbb{A}_3 \amalg 6\mathbb{A}_1$	18	$2_{II}^{-2}, 4_1^{+1}, 8_5^{-1}\ *$
		$\begin{pmatrix} 2\mathbb{A}_1 & 2\mathbb{A}_3 & (4\mathbb{A}_1)_I & 6\mathbb{A}_1 \\ & 4\mathbb{A}_1 & 6\mathbb{A}_1 & 8\mathbb{A}_1 \\ & & 2\mathbb{A}_1 & 2\mathbb{A}_3 \\ & & & 4\mathbb{A}_1 \end{pmatrix} \subset 4\mathbb{A}_3\ o$	18	$2_{II}^{-2}, 4_2^{-2}\ *$
		$((2\mathbb{A}_1, 2\mathbb{A}_1)_I, 4\mathbb{A}_1, 8\mathbb{A}_1) \subset 4\mathbb{A}_1 \amalg 4\mathbb{A}_3$	18	$2_{II}^{-2}, 4_2^{-2}\ *$
		$(2\mathbb{A}_1, 4\mathbb{A}_1, 4\mathbb{A}_1, 4\mathbb{A}_1) \subset 14\mathbb{A}_1$	18	$2_{II}^{+2}, 4_5^{-1}, 8_5^{-1}\ *$
		$(2\mathbb{A}_1, 4\mathbb{A}_1, 4\mathbb{A}_1, 4\mathbb{A}_1) \subset 2\mathbb{A}_3 \amalg 8\mathbb{A}_1$	18	$2_{II}^{+2}, 4_6^{+2}\ *$
		$(4\mathbb{A}_1, 4\mathbb{A}_1, 4\mathbb{A}_1, 4\mathbb{A}_1) \subset 16\mathbb{A}_1\ o$	18	$2_{II}^{+2}, 4_6^{+2}\ *$
		$\begin{pmatrix} 2\mathbb{A}_1 & (4\mathbb{A}_1)_I & (4\mathbb{A}_1)_I & (4\mathbb{A}_1)_I & 10\mathbb{A}_1 \\ & 2\mathbb{A}_1 & (4\mathbb{A}_1)_I & (4\mathbb{A}_1)_I & 10\mathbb{A}_1 \\ & & 2\mathbb{A}_1 & (4\mathbb{A}_1)_I & 10\mathbb{A}_1 \\ & & & 2\mathbb{A}_1 & 10\mathbb{A}_1 \\ & & & & 8\mathbb{A}_1 \end{pmatrix} \subset 16\mathbb{A}_1\ o$	19	$4_5^{+3}\ *$
		$\begin{pmatrix} 2\mathbb{A}_1 & (4\mathbb{A}_1)_I & 6\mathbb{A}_1 & 6\mathbb{A}_1 & 6\mathbb{A}_1 \\ & 2\mathbb{A}_1 & 6\mathbb{A}_1 & (4\mathbb{A}_1)_I & 6\mathbb{A}_1 \\ & & 4\mathbb{A}_1 & 6\mathbb{A}_1 & 8\mathbb{A}_1 \\ & & & 2\mathbb{A}_1 & 2\mathbb{A}_3 \\ & & & & 4\mathbb{A}_1 \end{pmatrix} \subset 8\mathbb{A}_1 \amalg 2\mathbb{A}_3$	19	$4_4^{-2}, 8_5^{-1}\ *$
		$\begin{pmatrix} 2\mathbb{A}_1 & (4\mathbb{A}_1)_I & 6\mathbb{A}_1 & (4\mathbb{A}_1)_I & 6\mathbb{A}_1 \\ & 2\mathbb{A}_1 & 2\mathbb{A}_3 & (4\mathbb{A}_1)_I & 6\mathbb{A}_1 \\ & & 4\mathbb{A}_1 & 6\mathbb{A}_1 & 8\mathbb{A}_1 \\ & & & 2\mathbb{A}_1 & 2\mathbb{A}_3 \\ & & & & 4\mathbb{A}_1 \end{pmatrix} \subset 2\mathbb{A}_1 \amalg 4\mathbb{A}_3\ o$	19	$4_5^{+3}\ *$

n	G	Deg	$\mathrm{rk}\, S$	q_S
9	$(C_2)^3$	$\begin{pmatrix} 2\mathbb{A}_1 & (4\mathbb{A}_1)_I & (4\mathbb{A}_1)_I & 6\mathbb{A}_1 & 10\mathbb{A}_1 \\ & 2\mathbb{A}_1 & (4\mathbb{A}_1)_I & 6\mathbb{A}_1 & 10\mathbb{A}_1 \\ & & 2\mathbb{A}_1 & 6\mathbb{A}_1 & 10\mathbb{A}_1 \\ & & & 4\mathbb{A}_1 & 4\mathbb{A}_3 \\ & & & & 8\mathbb{A}_1 \end{pmatrix} \subset 6\mathbb{A}_1 \amalg 4\mathbb{A}_3$	19	4_5^{+3} $*$
		$((2\mathbb{A}_1, 2\mathbb{A}_1)_{II}, 4\mathbb{A}_1, 4\mathbb{A}_1, 4\mathbb{A}_1) \subset 16\mathbb{A}_1$ o	19	$2_{II}^{-2}, 8_5^{-1}$ $*$
		$\begin{pmatrix} 2\mathbb{A}_1 & 2\mathbb{A}_3 & (4\mathbb{A}_1)_I & 6\mathbb{A}_1 & 6\mathbb{A}_1 \\ & 4\mathbb{A}_1 & 6\mathbb{A}_1 & 8\mathbb{A}_1 & 8\mathbb{A}_1 \\ & & 2\mathbb{A}_1 & 2\mathbb{A}_3 & 6\mathbb{A}_1 \\ & & & 4\mathbb{A}_1 & 8\mathbb{A}_1 \\ & & & & 4\mathbb{A}_1 \end{pmatrix} \subset 4\mathbb{A}_3 \amalg 4\mathbb{A}_1$ o	19	$2_{II}^{-2}, 8_5^{-1}$ $*$

Table 5: The List 1.

The list of cases when a degeneration of K3 with a symplectic automorphism group G_1 from Tables 1—4 has, actually the full symplectic automorphism group G_2 from Tables 1—4 which contains G_1, and $|G_1| < |G_2|$. The group G_2 has less orbits and less codimension of the degeneration than G_1.

$(\mathbf{n}=9,\ ((2\mathbb{A}_1, 2\mathbb{A}_1) \subset 4\mathbb{A}_1)_{II}) \Longleftarrow (\mathbf{n}=21, 4\mathbb{A}_1)$

$(\mathbf{n}=9,\ (8\mathbb{A}_1, 8\mathbb{A}_1) \subset 16\mathbb{A}_1) \Longleftarrow (\mathbf{n}=21, 16\mathbb{A}_1)$

$(\mathbf{n}=9,\ ((2\mathbb{A}_1, 2\mathbb{A}_1)_{II}, 4\mathbb{A}_1) \subset 8\mathbb{A}_1) \Longleftarrow (\mathbf{n}=21, (4\mathbb{A}_1, 4\mathbb{A}_1) \subset 8\mathbb{A}_1)$

$$(\mathbf{n}{=}9, \begin{pmatrix} 2\mathbb{A}_1 & (4\mathbb{A}_1)_I & (4\mathbb{A}_1)_I & (4\mathbb{A}_1)_I \\ & 2\mathbb{A}_1 & (4\mathbb{A}_1)_I & (4\mathbb{A}_1)_I \\ & & 2\mathbb{A}_1 & (4\mathbb{A}_1)_I \\ & & & 2\mathbb{A}_1 \end{pmatrix} \subset 8\mathbb{A}_1) \Longleftarrow (\mathbf{n}=40,\ 8\mathbb{A}_1)$$

$(\mathbf{n}=9,\ ((2\mathbb{A}_1, 2\mathbb{A}_1)_{II}, 4\mathbb{A}_1, 4\mathbb{A}_1) \subset 12\mathbb{A}_1) \Longleftarrow (\mathbf{n}=49,\ 12\mathbb{A}_1)$

$$(\mathbf{n}{=}9, \begin{pmatrix} 2\mathbb{A}_1 & 2\mathbb{A}_3 & (4\mathbb{A}_1)_I & 6\mathbb{A}_1 \\ & 4\mathbb{A}_1 & 6\mathbb{A}_1 & 8\mathbb{A}_1 \\ & & 2\mathbb{A}_1 & 2\mathbb{A}_3 \\ & & & 4\mathbb{A}_1 \end{pmatrix} \subset 4\mathbb{A}_3) \Longleftarrow (\mathbf{n}=22, (4\mathbb{A}_1, 8\mathbb{A}_1) \subset 4\mathbb{A}_3)$$

$(\mathbf{n}=9,\ (4\mathbb{A}_1, 4\mathbb{A}_1, 4\mathbb{A}_1, 4\mathbb{A}_1) \subset 16\mathbb{A}_1) \Longleftarrow (\mathbf{n}=39,\ 16\mathbb{A}_1)$

$$(\mathbf{n}{=}9, \begin{pmatrix} 2\mathbb{A}_1 & (4\mathbb{A}_1)_I & (4\mathbb{A}_1)_I & (4\mathbb{A}_1)_I & 10\mathbb{A}_1 \\ & 2\mathbb{A}_1 & (4\mathbb{A}_1)_I & (4\mathbb{A}_1)_I & 10\mathbb{A}_1 \\ & & 2\mathbb{A}_1 & (4\mathbb{A}_1)_I & 10\mathbb{A}_1 \\ & & & 2\mathbb{A}_1 & 10\mathbb{A}_1 \\ & & & & 8\mathbb{A}_1 \end{pmatrix} \subset 16\mathbb{A}_1) \Longleftarrow (\mathbf{n}=56,\ 16\mathbb{A}_1)$$

$$(\mathbf{n}{=}9,\ \begin{pmatrix} 2\mathbb{A}_1 & (4\mathbb{A}_1)_I & 6\mathbb{A}_1 & (4\mathbb{A}_1)_I & 6\mathbb{A}_1 \\ & 2\mathbb{A}_1 & 2\mathbb{A}_3 & (4\mathbb{A}_1)_I & 6\mathbb{A}_1 \\ & & 4\mathbb{A}_1 & 6\mathbb{A}_1 & 8\mathbb{A}_1 \\ & & & 2\mathbb{A}_1 & 2\mathbb{A}_3 \\ & & & & 4\mathbb{A}_1 \end{pmatrix} \subset 2\mathbb{A}_1 \amalg 4\mathbb{A}_3)$$

$$\Longleftarrow\ (\mathbf{n}=22,\ \begin{pmatrix} 2\mathbb{A}_1 & 6\mathbb{A}_1 & 10\mathbb{A}_1 \\ & 4\mathbb{A}_1 & 4\mathbb{A}_3 \\ & & 8\mathbb{A}_1 \end{pmatrix} \subset 2\mathbb{A}_1 \amalg 4\mathbb{A}_3)$$

$$(\mathbf{n}=9,\ ((2\mathbb{A}_1, 2\mathbb{A}_1)_{II}, 4\mathbb{A}_1, 4\mathbb{A}_1, 4\mathbb{A}_1) \subset 16\mathbb{A}_1) \Longleftarrow (\mathbf{n}=75,\ 16\mathbb{A}_1)$$

$$(\mathbf{n}=9,\ \begin{pmatrix} 2\mathbb{A}_1 & 2\mathbb{A}_3 & (4\mathbb{A}_1)_I & 6\mathbb{A}_1 & 6\mathbb{A}_1 \\ & 4\mathbb{A}_1 & 6\mathbb{A}_1 & 8\mathbb{A}_1 & 8\mathbb{A}_1 \\ & & 2\mathbb{A}_1 & 2\mathbb{A}_3 & 6\mathbb{A}_1 \\ & & & 4\mathbb{A}_1 & 8\mathbb{A}_1 \\ & & & & 4\mathbb{A}_1 \end{pmatrix} \subset 4\mathbb{A}_3 \amalg 4\mathbb{A}_1)$$

$$\Longleftarrow (\mathbf{n}=22,\ \begin{pmatrix} 4\mathbb{A}_1 & (8\mathbb{A}_1)_{II} & 12\mathbb{A}_1 \\ & 4\mathbb{A}_1 & 4\mathbb{A}_3 \\ & & 8\mathbb{A}_1 \end{pmatrix} \subset 4\mathbb{A}_1 \amalg 4\mathbb{A}_3)$$

$$(\mathbf{n}{=}10,\ (\mathbb{A}_1, \mathbb{A}_1) \subset 2\mathbb{A}_1)\ \Longleftarrow\ (\mathbf{n}=22, 2\mathbb{A}_1)$$

$$(\mathbf{n}{=}10,\ ((4\mathbb{A}_1, 4\mathbb{A}_1) \subset 8\mathbb{A}_1)_{II}) \Longleftarrow\ (\mathbf{n}=22, 8\mathbb{A}_1)$$

$$(\mathbf{n}{=}10,\ (\mathbb{A}_1, \mathbb{A}_1, (2\mathbb{A}_1)_I) \subset 4\mathbb{A}_1)\ \Longleftarrow (\mathbf{n}=39,\ 4\mathbb{A}_1)$$

$$(\mathbf{n}{=}10,\ (\mathbb{A}_1, \mathbb{A}_1, 4\mathbb{A}_1) \subset 6\mathbb{A}_1)\ \Longleftarrow\ (\mathbf{n}=22, (2\mathbb{A}_1, 4\mathbb{A}_1) \subset 6\mathbb{A}_1)$$

$$(\mathbf{n}{=}10,\ (\mathbb{A}_1, \mathbb{A}_1, 8\mathbb{A}_1) \subset 10\mathbb{A}_1)\ \Longleftarrow\ (\mathbf{n}=22, (2\mathbb{A}_1, 8\mathbb{A}_1) \subset 10\mathbb{A}_1)$$

$$(\mathbf{n}{=}10,\ \begin{pmatrix} (2\mathbb{A}_1)_I & (6\mathbb{A}_1)_I & (6\mathbb{A}_1)_I \\ & 4\mathbb{A}_1 & (8\mathbb{A}_1)_{II} \\ & & 4\mathbb{A}_1 \end{pmatrix} \subset 10\mathbb{A}_1)$$
$$\Longleftarrow\ (\mathbf{n}=22, (2\mathbb{A}_1, 8\mathbb{A}_1) \subset 10\mathbb{A}_1)$$

$$(\mathbf{n}{=}10,\ \begin{pmatrix} (2\mathbb{A}_1)_{II} & 6\mathbb{A}_1 & 6\mathbb{A}_1 \\ & 4\mathbb{A}_1 & (8\mathbb{A}_1)_{II} \\ & & 4\mathbb{A}_1 \end{pmatrix} \subset 10\mathbb{A}_1)$$

$$\Longleftarrow\ (\mathbf{n}=22, (2\mathbb{A}_1, 8\mathbb{A}_1) \subset 10\mathbb{A}_1)$$

$$(\mathbf{n}{=}10,\ \begin{pmatrix} 4\mathbb{A}_1 & (8\mathbb{A}_1)_{I} & (8\mathbb{A}_1)_{II} \\ & 4\mathbb{A}_1 & (8\mathbb{A}_1)_{I} \\ & & 4\mathbb{A}_1 \end{pmatrix} \subset 12\mathbb{A}_1)$$

$$\Longleftarrow\ (\mathbf{n}=22, (4\mathbb{A}_1, 8\mathbb{A}_1) \subset 12\mathbb{A}_1)$$

$$(\mathbf{n}{=}10,\ \begin{pmatrix} 4\mathbb{A}_1 & 4\mathbb{A}_2 & (8\mathbb{A}_1)_{II} \\ & 4\mathbb{A}_1 & 4\mathbb{A}_2 \\ & & 4\mathbb{A}_1 \end{pmatrix} \subset 4\mathbb{A}_3)$$

$$\Longleftarrow\ (\mathbf{n}=22, (4\mathbb{A}_1, 8\mathbb{A}_1) \subset 4\mathbb{A}_3)$$

$$(\mathbf{n}{=}10,\ \begin{pmatrix} 4\mathbb{A}_1 & (8\mathbb{A}_1)_{II} & 12\mathbb{A}_1 \\ & 4\mathbb{A}_1 & 12\mathbb{A}_1 \\ & & 8\mathbb{A}_1 \end{pmatrix} \subset 16\mathbb{A}_1) \Longleftarrow (\mathbf{n}=39,\ 16\mathbb{A}_1)$$

$$(\mathbf{n}{=}10,\ \begin{pmatrix} \mathbb{A}_1 & 2\mathbb{A}_1 & 3\mathbb{A}_1 & 5\mathbb{A}_1 \\ & \mathbb{A}_1 & 3\mathbb{A}_1 & 5\mathbb{A}_1 \\ & & (2\mathbb{A}_1)_I & (6\mathbb{A}_1)_I \\ & & & 4\mathbb{A}_1 \end{pmatrix} \subset 8\mathbb{A}_1) \Longleftarrow (\mathbf{n}=56,\ 8\mathbb{A}_1)$$

$$(\mathbf{n}=10,\ (\mathbb{A}_1, \mathbb{A}_1, (2\mathbb{A}_1)_I, 8\mathbb{A}_1) \subset 12\mathbb{A}_1) \Longleftarrow (\mathbf{n}=65,\ 12\mathbb{A}_1)$$

$$(\mathbf{n}{=}10,\ \begin{pmatrix} \mathbb{A}_1 & 2\mathbb{A}_1 & 5\mathbb{A}_1 & 5\mathbb{A}_1 \\ & \mathbb{A}_1 & 5\mathbb{A}_1 & 5\mathbb{A}_1 \\ & & 4\mathbb{A}_1 & (8\mathbb{A}_1)_I \\ & & & 4\mathbb{A}_1 \end{pmatrix} \subset 10\mathbb{A}_1)$$

$$\Longleftarrow (\mathbf{n}=22,\ (2\mathbb{A}_1, (4\mathbb{A}_1, 4\mathbb{A}_1)_{II}) \subset 10\mathbb{A}_1)$$

$$(\mathbf{n}=10,\ (\mathbb{A}_1,\ \mathbb{A}_1,\ 4\mathbb{A}_1,\ 8\mathbb{A}_1) \subset 14\mathbb{A}_1)$$

$$\Longleftarrow\ (\mathbf{n}=22,\ (2\mathbb{A}_1, 4\mathbb{A}_1, 8\mathbb{A}_1) \subset 14\mathbb{A}_1)$$

$$(\mathbf{n}=10,\ (\mathbb{A}_1,\ \mathbb{A}_1,\ 4\mathbb{A}_1,\ 8\mathbb{A}_1) \subset 2\mathbb{A}_1 \amalg 4\mathbb{A}_3),$$

$$\Longleftarrow\ (\mathbf{n}=22,\ \begin{pmatrix} 2\mathbb{A}_1 & 6\mathbb{A}_1 & 10\mathbb{A}_1 \\ & 4\mathbb{A}_1 & 4\mathbb{A}_3 \\ & & 8\mathbb{A}_1 \end{pmatrix} \subset 2\mathbb{A}_1 \amalg 4\mathbb{A}_3)$$

$$(\mathbf{n}{=}10,\ \begin{pmatrix} \mathbb{A}_1 & \mathbb{A}_3 & 3\mathbb{A}_1 & 5\mathbb{A}_1 \\ & (2\mathbb{A}_1)_{II} & 4\mathbb{A}_1 & 6\mathbb{A}_1 \\ & & (2\mathbb{A}_1)_I & 2\mathbb{A}_3 \\ & & & 4\mathbb{A}_1 \end{pmatrix} \subset 3\mathbb{A}_3)$$

$$\Longleftarrow (\mathbf{n}=34,\ (3\mathbb{A}_1, (6\mathbb{A}_1)_{II}) \subset 3\mathbb{A}_3))$$

$$(\mathbf{n}{=}10,\ \begin{pmatrix} \mathbb{A}_1 & 3\mathbb{A}_1 & 5\mathbb{A}_1 & 9\mathbb{A}_1 \\ & (2\mathbb{A}_1)_I & (6\mathbb{A}_1)_I & 10\mathbb{A}_1 \\ & & 4\mathbb{A}_1 & 12\mathbb{A}_1 \\ & & & 8\mathbb{A}_1 \end{pmatrix} \subset 15\mathbb{A}_1)$$

$$\Longleftarrow (\mathbf{n}=34,\ (3\mathbb{A}_1, 12\mathbb{A}_1)) \subset 15\mathbb{A}_1))$$

$(\mathbf{n}{=}10,\ (\mathbb{A}_1,\ 4\mathbb{A}_1,\ 4\mathbb{A}_1,\ 2\mathbb{A}_2) \subset \mathbb{A}_1 \amalg 6\mathbb{A}_2) \Longleftarrow (\mathbf{n}=34,\ (\mathbb{A}_1, 6\mathbb{A}_2) \subset \mathbb{A}_1 \amalg 6\mathbb{A}_2))$

$$(\mathbf{n}{=}10,\ \begin{pmatrix} (2\mathbb{A}_1)_I & 4\mathbb{A}_1 & (6\mathbb{A}_1)_I & (6\mathbb{A}_1)_I \\ & (2\mathbb{A}_1)_{II} & 6\mathbb{A}_1 & 6\mathbb{A}_1 \\ & & 4\mathbb{A}_1 & (8\mathbb{A}_1)_{II} \\ & & & 4\mathbb{A}_1 \end{pmatrix} \subset 12\mathbb{A}_1)$$

$$\Longleftarrow (\mathbf{n}=65,\ 12\mathbb{A}_1)$$

$$(\mathbf{n}{=}10,\ \begin{pmatrix} (2\mathbb{A}_1)_I & (6\mathbb{A}_1)_{II} & (6\mathbb{A}_1)_I & (6\mathbb{A}_1)_I \\ & 4\mathbb{A}_1 & (8\mathbb{A}_1)_I & (8\mathbb{A}_1)_I \\ & & 4\mathbb{A}_1 & (8\mathbb{A}_1)_{II} \\ & & & 4\mathbb{A}_1 \end{pmatrix} \subset 14\mathbb{A}_1)$$

$$\Longleftarrow\ (\mathbf{n}=22,\ (2\mathbb{A}_1, 4\mathbb{A}_1, 8\mathbb{A}_1) \subset 14\mathbb{A}_1)$$

$$(\mathbf{n}{=}10,\ \begin{pmatrix} (2\mathbb{A}_1)_{II} & 6\mathbb{A}_1 & 6\mathbb{A}_1 & 6\mathbb{A}_1 \\ & 4\mathbb{A}_1 & (8\mathbb{A}_1)_I & (8\mathbb{A}_1)_I \\ & & 4\mathbb{A}_1 & (8\mathbb{A}_1)_{II} \\ & & & 4\mathbb{A}_1 \end{pmatrix} \subset 14\mathbb{A}_1)$$

$$\Longleftarrow\ (\mathbf{n}=22,\ (2\mathbb{A}_1, 4\mathbb{A}_1, 8\mathbb{A}_1) \subset 14\mathbb{A}_1)$$

$$(\mathbf{n}{=}10,\ \begin{pmatrix} (2\mathbb{A}_1)_I & (6\mathbb{A}_1)_I & (6\mathbb{A}_1)_I & (6\mathbb{A}_1)_I \\ & 4\mathbb{A}_1 & 4\mathbb{A}_2 & (8\mathbb{A}_1)_{II} \\ & & 4\mathbb{A}_1 & 4\mathbb{A}_2 \\ & & & 4\mathbb{A}_1 \end{pmatrix} \subset 2\mathbb{A}_1 \amalg 4\mathbb{A}_3),$$

$$(\mathbf{n}{=}10,\ \begin{pmatrix} (2\mathbb{A}_1)_{II} & 6\mathbb{A}_1 & 6\mathbb{A}_1 & 6\mathbb{A}_1 \\ & 4\mathbb{A}_1 & 4\mathbb{A}_2 & (8\mathbb{A}_1)_{II} \\ & & 4\mathbb{A}_1 & 4\mathbb{A}_2 \\ & & & 4\mathbb{A}_1 \end{pmatrix} \subset 2\mathbb{A}_1 \amalg 4\mathbb{A}_3)$$

$$\Longleftarrow\ (\mathbf{n}=22,\ \begin{pmatrix} 2\mathbb{A}_1 & 6\mathbb{A}_1 & 10\mathbb{A}_1 \\ & 4\mathbb{A}_1 & 4\mathbb{A}_3 \\ & & 8\mathbb{A}_1 \end{pmatrix} \subset 2\mathbb{A}_1 \amalg 4\mathbb{A}_3)$$

$$(\mathbf{n}{=}10,\ \begin{pmatrix} (2\mathbb{A}_1)_I & 2\mathbb{A}_3 & (6\mathbb{A}_1)_I & 10\mathbb{A}_1 \\ & 4\mathbb{A}_1 & (8\mathbb{A}_1)_I & 12\mathbb{A}_1 \\ & & 4\mathbb{A}_1 & 4\mathbb{A}_3 \\ & & & 8\mathbb{A}_1 \end{pmatrix} \subset 6\mathbb{A}_3)$$

$$\Longleftarrow (\mathbf{n}=34,\ ((6\mathbb{A}_1)_I, 12\mathbb{A}_1) \subset 6\mathbb{A}_3)$$

$$(\mathbf{n}{=}10,\ \begin{pmatrix} 4\mathbb{A}_1 & (8\mathbb{A}_1)_{II} & (8\mathbb{A}_1)_I & (8\mathbb{A}_1)_I \\ & 4\mathbb{A}_1 & (8\mathbb{A}_1)_I & (8\mathbb{A}_1)_I \\ & & 4\mathbb{A}_1 & (8\mathbb{A}_1)_{II} \\ & & & 4\mathbb{A}_1 \end{pmatrix} \subset 16\mathbb{A}_1) \Longleftarrow (\mathbf{n}=56,\ 16\mathbb{A}_1)$$

$$(\mathbf{n}{=}10,\ \begin{pmatrix} 4\mathbb{A}_1 & (8\mathbb{A}_1)_I & (8\mathbb{A}_1)_I & (8\mathbb{A}_1)_I \\ & 4\mathbb{A}_1 & 4\mathbb{A}_2 & (8\mathbb{A}_1)_{II} \\ & & 4\mathbb{A}_1 & 4\mathbb{A}_2 \\ & & & 4\mathbb{A}_1 \end{pmatrix} \subset 4\mathbb{A}_1 \amalg 4\mathbb{A}_3)$$

$$\Longleftarrow (\mathbf{n}=22,\ \begin{pmatrix} 4\mathbb{A}_1 & (8\mathbb{A}_1)_{II} & 12\mathbb{A}_1 \\ & 4\mathbb{A}_1 & 4\mathbb{A}_3 \\ & & 8\mathbb{A}_1 \end{pmatrix} \subset 4\mathbb{A}_1 \amalg 4\mathbb{A}_3)$$

$$(\mathbf{n}{=}10,\ \begin{pmatrix} 4\mathbb{A}_1 & (8\mathbb{A}_1)_I & (8\mathbb{A}_1)_I & 4\mathbb{A}_1 \amalg 2\mathbb{A}_2 \\ & 4\mathbb{A}_1 & 4\mathbb{A}_2 & 4\mathbb{A}_1 \amalg 2\mathbb{A}_2 \\ & & 4\mathbb{A}_1 & 4\mathbb{A}_1 \amalg 2\mathbb{A}_2 \\ & & & 2\mathbb{A}_2 \end{pmatrix} \subset 4\mathbb{A}_1 \amalg 6\mathbb{A}_2)$$

$$\Longleftarrow (\mathbf{n} = 34,\ (4\mathbb{A}_1, 6\mathbb{A}_2) \subset 4\mathbb{A}_1 \amalg 6\mathbb{A}_2)$$

$$(\mathbf{n} = 12,\ (8\mathbb{A}_1, 8\mathbb{A}_1) \subset 16\mathbb{A}_1) \Longleftarrow (\mathbf{n} = 75,\ 16\mathbb{A}_1)$$

$$(\mathbf{n} = 12,\ (\mathbb{A}_2, \mathbb{A}_2) \subset 2\mathbb{A}_2) \Longleftarrow (\mathbf{n} = 26,\ 2\mathbb{A}_2)$$

$$(\mathbf{n} = 17,\ (\mathbb{A}_1, \mathbb{A}_1) \subset 2\mathbb{A}_1) \Longleftarrow (\mathbf{n} = 34,\ 2\mathbb{A}_1)$$

$$(\mathbf{n} = 17,\ (\mathbb{A}_1, 3\mathbb{A}_1) \subset 4\mathbb{A}_1) \Longleftarrow (\mathbf{n} = 49,\ 4\mathbb{A}_1)$$

$$(\mathbf{n} = 17,\ (4\mathbb{A}_1, 4\mathbb{A}_1) \subset 8\mathbb{A}_1) \Longleftarrow (\mathbf{n} = 34,\ 8\mathbb{A}_1)$$

$$(\mathbf{n} = 17,\ (4\mathbb{A}_1, 12\mathbb{A}_1) \subset 16\mathbb{A}_1) \Longleftarrow (\mathbf{n} = 49,\ 16\mathbb{A}_1)$$

$$(\mathbf{n} = 17,\ (6\mathbb{A}_1, 6\mathbb{A}_1) \subset 12\mathbb{A}_1) \Longleftarrow (\mathbf{n} = 49,\ 12\mathbb{A}_1)$$

$$(\mathbf{n} = 17,\ (6\mathbb{A}_1, 6\mathbb{A}_1) \subset 6\mathbb{A}_2) \Longleftarrow (\mathbf{n} = 34,\ 8\mathbb{A}_1)$$

$$(\mathbf{n} = 17,\ (\mathbb{A}_1, \mathbb{A}_1, \mathbb{A}_1) \subset 3\mathbb{A}_1) \Longleftarrow (\mathbf{n} = 61,\ 3\mathbb{A}_1)$$

$$(\mathbf{n} = 17,\ (\mathbb{A}_1, \mathbb{A}_1, 4\mathbb{A}_1) \subset 6\mathbb{A}_1) \Longleftarrow (\mathbf{n} = 34,\ (2\mathbb{A}_1, 4\mathbb{A}_1) \subset 6\mathbb{A}_1)$$

$$(\mathbf{n} = 17,\ (\mathbb{A}_1, \mathbb{A}_1, 6\mathbb{A}_1) \subset 8\mathbb{A}_1) \Longleftarrow (\mathbf{n} = 34,\ (2\mathbb{A}_1, (6\mathbb{A}_1)_{II}) \subset 8\mathbb{A}_1)$$

$$(\mathbf{n} = 17,\ (\mathbb{A}_1, \mathbb{A}_1, 12\mathbb{A}_1) \subset 14\mathbb{A}_1) \Longleftarrow (\mathbf{n} = 34,\ (2\mathbb{A}_1, 12\mathbb{A}_1) \subset 14\mathbb{A}_1)$$

$$(\mathbf{n} = 17,\ (\mathbb{A}_1, 3\mathbb{A}_1, 4\mathbb{A}_1) \subset 8\mathbb{A}_1) \Longleftarrow (\mathbf{n} = 65,\ 8\mathbb{A}_1)$$

$$(\mathbf{n} = 17,\ (\mathbb{A}_1, 3\mathbb{A}_1, 12\mathbb{A}_1) \subset 16\mathbb{A}_1) \Longleftarrow (\mathbf{n} = 75,\ 16\mathbb{A}_1)$$

$$(\mathbf{n} = 17,\ (\mathbb{A}_1, 4\mathbb{A}_1, 4\mathbb{A}_1) \subset 9\mathbb{A}_1) \Longleftarrow (\mathbf{n} = 34,\ (\mathbb{A}_1, 8\mathbb{A}_1) \subset 9\mathbb{A}_1)$$

$$(\mathbf{n} = 17,\ (\mathbb{A}_1, 6\mathbb{A}_1, 6\mathbb{A}_1) \subset \mathbb{A}_1 \amalg 6\mathbb{A}_2) \Longleftarrow (\mathbf{n} = 34,\ (\mathbb{A}_1, 6\mathbb{A}_2) \subset \mathbb{A}_1 \amalg 6\mathbb{A}_2)$$

$$(\mathbf{n} = 17,\ (3\mathbb{A}_1, 4\mathbb{A}_1, 4\mathbb{A}_1) \subset 11\mathbb{A}_1) \Longleftarrow (\mathbf{n} = 34,\ (3\mathbb{A}_1, 8\mathbb{A}_1) \subset 11\mathbb{A}_1)$$

$$(\mathbf{n}{=}17,\ \begin{pmatrix} 4\mathbb{A}_1 & 4\mathbb{A}_2 & 8\mathbb{A}_1 \\ & 4\mathbb{A}_1 & 4\mathbb{A}_2 \\ & & 4\mathbb{A}_1 \end{pmatrix} \subset 4\mathbb{A}_3) \ \Longleftarrow (\mathbf{n} = 34,\ (4\mathbb{A}_1, 8\mathbb{A}_1) \subset 4\mathbb{A}_3)$$

$(\mathbf{n}=17,\ (4\mathbb{A}_1,4\mathbb{A}_1,6\mathbb{A}_1)\subset 14\mathbb{A}_1)\ \Longleftarrow (\mathbf{n}=34,\ ((6\mathbb{A}_1)_I,8\mathbb{A}_1)\subset 14\mathbb{A}_1)$

$(\mathbf{n}=17,\ (4\mathbb{A}_1,6\mathbb{A}_1,6\mathbb{A}_1)\subset 16\mathbb{A}_1)\Longleftarrow (\mathbf{n}=75,\ 16\mathbb{A}_1)$

$(\mathbf{n}=17,\ (4\mathbb{A}_1,6\mathbb{A}_1,6\mathbb{A}_1)\subset 4\mathbb{A}_1\amalg 6\mathbb{A}_2)\Longleftarrow (\mathbf{n}=34,\ (4\mathbb{A}_1,6\mathbb{A}_2)\subset 4\mathbb{A}_1\amalg 6\mathbb{A}_2)$

$(\mathbf{n}=21,\ (4\mathbb{A}_1,4\mathbb{A}_1,4\mathbb{A}_1)\subset 12\mathbb{A}_1)\Longleftarrow (\mathbf{n}=49,\ 12\mathbb{A}_1)$

$(\mathbf{n}=21,\ (4\mathbb{A}_1,4\mathbb{A}_1,4\mathbb{A}_1,4\mathbb{A}_1)\subset 16\mathbb{A}_1)\Longleftarrow (\mathbf{n}=75,\ 16\mathbb{A}_1)$

$(\mathbf{n}=22,\ (2\mathbb{A}_1,2\mathbb{A}_1)\subset 4\mathbb{A}_1)\Longleftarrow (\mathbf{n}=39,\ 4\mathbb{A}_1)$

$(\mathbf{n}=22,\ ((4\mathbb{A}_1,4\mathbb{A}_1)\subset 8\mathbb{A}_1)_I)\Longleftarrow (\mathbf{n}=40,\ 8\mathbb{A}_1)$

$(\mathbf{n}=22,\ (8\mathbb{A}_1,8\mathbb{A}_1)\subset 16\mathbb{A}_1)\ \Longleftarrow (\mathbf{n}=39,\ 16\mathbb{A}_1)$

$(\mathbf{n}=22,\ (2\mathbb{A}_1,2\mathbb{A}_1,4\mathbb{A}_1)\subset 8\mathbb{A}_1)\Longleftarrow (\mathbf{n}=56,\ 8\mathbb{A}_1)$

$(\mathbf{n}=22,\ (2\mathbb{A}_1,2\mathbb{A}_1,8\mathbb{A}_1)\subset 12\mathbb{A}_1)\Longleftarrow (\mathbf{n}=65,\ 12\mathbb{A}_1)$

$(\mathbf{n}=22,\ ((4\mathbb{A}_1,4\mathbb{A}_1)_I,8\mathbb{A}_1)\subset 16\mathbb{A}_1)\Longleftarrow (\mathbf{n}=56,\ 16\mathbb{A}_1)$

$(\mathbf{n}=34,\ (\mathbb{A}_1,\mathbb{A}_1)\subset 2\mathbb{A}_1)\Longleftarrow (\mathbf{n}=51,\ 2\mathbb{A}_1)$

$(\mathbf{n}=34,\ (\mathbb{A}_1,2\mathbb{A}_1)\subset 3\mathbb{A}_1)\Longleftarrow (\mathbf{n}=61,\ 3\mathbb{A}_1)$

$(\mathbf{n}=34,\ (\mathbb{A}_1,3\mathbb{A}_1)\subset 4\mathbb{A}_1)\Longleftarrow (\mathbf{n}=65,\ 4\mathbb{A}_1)$

$(\mathbf{n}=34,\ (4\mathbb{A}_1,4\mathbb{A}_1)\subset 8\mathbb{A}_1)\Longleftarrow (\mathbf{n}=51,\ 8\mathbb{A}_1)$

$(\mathbf{n}=34,\ (4\mathbb{A}_1,8\mathbb{A}_1)\subset 12\mathbb{A}_1)\Longleftarrow (\mathbf{n}=61,\ 12\mathbb{A}_1)$

$(\mathbf{n}=34,\ (4\mathbb{A}_1,12\mathbb{A}_1)\subset 16\mathbb{A}_1)\Longleftarrow (\mathbf{n}=65,\ 16\mathbb{A}_1)$

$(\mathbf{n}=34,\ ((6\mathbb{A}_1)_I,(6\mathbb{A}_1)_{II})\subset 12\mathbb{A}_1)\Longleftarrow (\mathbf{n}=65,\ 12\mathbb{A}_1)$

$(\mathbf{n}=39,\ (4\mathbb{A}_1,4\mathbb{A}_1)\subset 8\mathbb{A}_1)\Longleftarrow (\mathbf{n}=56,\ 8\mathbb{A}_1)$

$(\mathbf{n}=39,\ (4\mathbb{A}_1,8\mathbb{A}_1)\subset 12\mathbb{A}_1)\Longleftarrow (\mathbf{n}=65,\ 12\mathbb{A}_1)$

$(\mathbf{n}=39,\ (8\mathbb{A}_1,8\mathbb{A}_1)\subset 16\mathbb{A}_1)\Longleftarrow (\mathbf{n}=75,\ 16\mathbb{A}_1)$

$(\mathbf{n}=40,\ (8\mathbb{A}_1,8\mathbb{A}_1)\subset 16\mathbb{A}_1)\Longleftarrow (\mathbf{n}=56,\ 16\mathbb{A}_1)$

$(\mathbf{n}=49,\ (4\mathbb{A}_1,4\mathbb{A}_1)\subset 8\mathbb{A}_1)\Longleftarrow (\mathbf{n}=65,\ 8\mathbb{A}_1)$

$(\mathbf{n}=49,\ (4\mathbb{A}_1,12\mathbb{A}_1)\subset 16\mathbb{A}_1)\Longleftarrow (\mathbf{n}=75,\ 16\mathbb{A}_1)$

2 Classification of Picard lattices of K3 surfaces.

Actually, classification of Tables 1—4 contains the important classification of Picard lattices of K3 surfaces.

Let S_X be the Picard lattice of a Kählerian K3 surface X, and $S_X < 0$, is negative definite. For algebraic K3, instead of S_X one can take $(h)^\perp_{S_X}$ where $h \in S_X$ is a numerically effective element with $h^2 > 0$.

Let $G = Aut(X)_0$ be the full symplectic automorphism group of X (it is finite) and let $E_1, \dots E_k$ are all non-singular irreducible rational curves on X. Then

$$S = [(S)_G = (S_X)_G, cl(E_1), \dots cl(E_k)]_{pr} \subset S_X$$

gives the most important part of the Picard lattice S_X: (*the main part of the Picard lattice S_X or MS_X.* The remaining part $S \subset S_X$ of S_X gives *exotic* part of S_X or (*exotic $ES_X = (S)^\perp_{S_X}$ part of the Picard lattice S_X*). Classification of possible $S = MS_X$ is the most important.

Obviously, **the lattice $S = MS_X$ is one of lattices of Tables 1—4 (if $G > D_6$).** The only difference is that *the group G must be the maximal symplectic automorphism group of K3 for S.*

From the point of view of abstract lattices, $e_1 = cl(E_1), \ldots, e_k = cl(E_k)$ give the basis of the root system of (-2)-roots of the lattice S, and

$$G|S = \{g \in O(S) \mid g(\{e_1, ..., e_k\}) = \{e_1, ..., e_k\}),\ g|(S^*/S) = id\}.$$

The sublattice $S \subset S_X$ is the most important part of the Picard lattice: $S = MS_X$.

Another characterization of $S = MS_X$ is as follows. Let

$$H(S_X) = \{g \in O(S_X) \mid g|A(S_X) = (S_X)^*/S_X = identity\}.$$

Then $S = MS_X = (S_X)_{H(S_X)}$. Also $H(S_X) = W^{(-2)}(S_X) \rtimes G$.

For some two cases of Tables 1—4, lattices $S_1 \cong S_2$ can be isomorphic, but their symplectic groups $G_1 \subset G_2$ are subgroups of one another only of different orders $|G_1| < |G_2|$.

All pairs with isomorphic lattices S, but subgroups $G_1 \subset G_2$ $|G_1| < |G_2|$ are given in Table 5. **The case (S, G_1) is denoted by o (old) in Tables 1—4. Thus, for the classification of Picard lattices, we can delete the case (S, G_1).** But, this cases is important as itself: If K3 surface X has the symplectic automorphism group G_1 and the degeneration S, then the full symplectic automorphism group of X is larger, it is G_2 with $|G_1| < |G_2|$. The group G_2 has less orbits and less codimension of the degeneration than G_1. Therefore, the Table 5 is very interesting and important.

Exactly that happens for the case of *Kummer surfaces:* $|G_1| = 1$, $G_2 = (C_2)^4$, $S = [16\mathbb{A}_1]_{pr}$: Kummer surfaces give the degeneration of the type $16\mathbb{A}_1$ (or the codimension 16) of K3 with trivial symplectic automorphism group C_1, when the symplectic automorphism group becomes $(C_2)^4$. In 1974, I showed that K3 with 16 (sixteen) $\mathbb{P}^1$ and Dynkin diagram $16\mathbb{A}_1$ is Kummer.

Thus, finally, we get

Theorem. *The classification of Picard lattices $S = MS_X$ of K3 surfaces X with finite symplectic automorphism groups which are sufficiently large (larger than D_6, C_4, $(C_2)^2$, C_3, C_2 and C_1) and with at least one (-2)-curve are given in Tables 1—4 in lines which are not denoted by the o.*

3 Concluding remarks.

I hope to consider the remaining (small) symplectic automorphism groups D_6, C_4, $(C_2)^2$, C_3, C_2, C_1 later. Cases of D_6 and C_4 were recently considered in my papers [Nik14], [Nik15] of 2019. Now I consider the case of $(C_2)^2$. For this case, degenerations can be of codimension $t = 1, 2, \ldots, 6, 7$. There are many cases, but I hope to consider them soon.

Now, for the few remaining groups we have:

Theorem. *If the Picard lattice $S = MS_X$ of K3 surface X with at least one (-2)-curve is different from given in lines of Tables 1—4 and tables of [Nik14] for D_6 and tables of [Nik15] for C_4 which are not denoted by the sign o (for example, if its genus is different), then the symplectic automorphism group of X is small: it is one of groups $(C_2)^2$, C_3, C_2 or C_1. I hope to consider them later.*

Methods.

1) Markings by negative definite even unimodular lattices; for K3, it is enough to use Niemeier lattices N of rank 24. Their roots ($\alpha \in N$ with $\alpha^2 = -2$) sublattices are

$$N^{(2)} = [\Delta(N)] =$$

$$(1)\ D_{24},\ (2)\ D_{16} \oplus E_8,\ (3)\ 3E_8,\ (4)\ A_{24},\ (5)\ 2D_{12},$$

$$(6)\ A_{17} \oplus E_7,\ (7)\ D_{10} \oplus 2E_7,\ (8)\ A_{15} \oplus D_9,\ (9)\ 3D_8,$$

$$(10)\ 2A_{12},\ (11)\ A_{11} \oplus D_7 \oplus E_6,\ (12)\ 4E_6,\ (13)\ 2A_9 \oplus D_6,$$

$$(14)\ 4D_6,\ (15)\ 3A_8,\ (16)\ 2A_7 \oplus 2D_5,\ (17)\ 4A_6,\ (18)\ 4A_5 \oplus D_4,\ (19)\ 6D_4,$$

$$(20)\ 6A_4,\ (21)\ 8A_3,\ (22)\ 12A_2,\ (23)\ 24A_1$$

give 23 Niemeier lattices N_j. The last is Leech lattice (24) with $N^{(2)} = \{0\}$ which has no roots. Further, $N(R)$ denotes the Niemeier lattice with the root system R. We fix the basis $P(N)$ of the root system $\Delta(N)$ of N. By $A(N) \subset O(N)$ we denote the subgroup of the group of automorphisms of N which preserves (permutes) the basis $P(N)$.

The action of G on K3 can be modeled by $G \subset A(N)$ and its t orbits $G(r_1), \ldots, G(r_t)$ of roots $r_1, \ldots, r_t \in P(N)$. The lattice

$$S = [N_G, G(r_1), \ldots, G(r_t)]_{pr} = [N_G, r_1, \ldots, r_t]_{pr} \subset N.$$

It gives a degeneration of K3 with G of codimension t we are looking for iff S has a primitive embedding to the cohomology lattice $L_{K3} = H^2(X, \mathbb{Z})$ *which is an even unimodular lattice of signature* $(3, 19)$.

2) We need results about existence of primitive embeddings of lattices into even unimodular lattices.

Both this methods and results were suggested and developed in my papers:

Nikulin, Finite automorphism groups of Kähler K3 surfaces, *English transl. in* Trans. Moscow Math. Soc. ***V. 38*** *(1980), 71–135.*

Nikulin, Integral symmetric bilinear forms and some of their geometric applications, *English transl. in Math. USSR Izv.* ***14*** *(1980), no. 1, 103–167.*

3) We need classification of possible abstract finite symplectic automorphism groups of K3. Abelian such groups were classified in my paper above. Non-Abelian were classified by
Sh.Mukai, Finite groups of automorphisms of K3 surfaces and the Mathieu group, *Invent. math.* ***94****, (1988), 183–221.*

4) We use computer. We write programs.

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