Problem from CFT in genus one

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1 Motivation (for physicists)

In the $(2, 5)$ minimal model on complex curves, the Virasoro algebra has two irreducible representations, the vacuum respresentation and another representation with lowest weight $h = -1/5$.

To this representation corresponds a non-holomorphic field Φ. Locally it admits a splitting $\Phi(z, \bar{z}) = \varphi_{\text{hol}}(z) \otimes \overline{\varphi_{\text{hol}}}(\bar{z})$, where $\varphi(z) = \varphi_{\text{hol}}(z)$ is a holomorphic local primary field of that weight, and the second factor is its complex conjugate.

For $N \geq 2$, N-point functions involving Φ depend on the N positions on the curve, whereas N-point functions involving $\varphi(z)$ live on a cover of the curve. For $N = 2$ on a torus, translation invariance yields a function on the universal cover of a torus with one puncture. $\Psi(z) := \langle\varphi(z)\varphi(0)\rangle$ satisfies a third order ODE in z and yields a 3-dimensional representation of its monodromy group.

[arXiv:1801.08387]

2 Problem

Solve the OPE

$$\frac{25}{12}\Psi''' - \wp\Psi' + \frac{1}{5}\wp'\Psi = 0 \tag{1}$$

for Ψ. Here $\wp = \wp(z|\tau)$ and the dash denotes differentiation w.r.t. z.

3 Alternative formulation of the problem

Set $x = \wp$, $y^2 = p(x)$ and $dz = dx/y$ with $p(x) = 4\left(x^3 - \frac{\pi^4}{3}E_4 x - \frac{2}{27}\pi^6 E_6\right)$, where E_4, E_6 are the Eisenstein series in τ. Find the kernel of

$$p\frac{d^3}{dx^3} + \frac{3}{2}p'\frac{d^2}{dx^2} + \frac{12}{25}p''\frac{d}{dx} + \frac{12}{125}\,. \tag{2}$$